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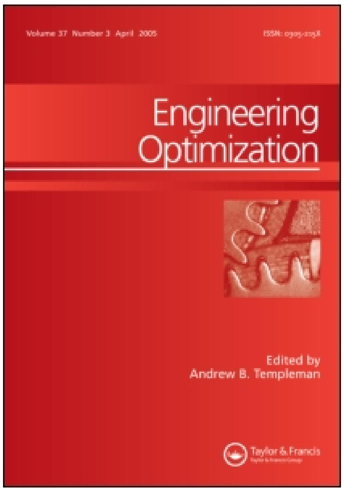

Engineering Optimization
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713641621

Assessing product family design from an end-of-life perspective

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First published on: 11 October 2010

To cite this Article Kwak, Minjung and Kim, Harrison M.(2010) 'Assessing product family design from an end-of-life perspective', Engineering Optimization,, First published on: 11 October 2010 (iFirst)
To link to this Article: DOI: 10.1080/0305215X.2010.482990
URL: http://dx.doi.org/10.1080/0305215X.2010.482990

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Engineering Optimization
iFirst, 2010, 1–23

Assessing product family design from an end-of-life perspective

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(*Received 11 January 2010; final version received 15 March 2010*)

Product family design via component sharing is a widely practiced approach for offering sufficient variety to the market in an economical way. When discussing product family design, most previous research has focused on its benefits in the design and manufacturing stages. This article highlights another important aspect of product family design: the impact of component sharing on end-of-life management. This article presents a quantitative model for assessing product family design from an end-of-life perspective. Using mixed integer programming, the developed model identifies an optimal strategy for managing product take-back and end-of-life recovery, thereby assessing the product family design in terms of its profitability in end-of-life management. Especially, the model incorporates increased component interchangeability by component sharing. A design study of a smart phone family is presented, as an illustration, and the results show that the model can assess profitability of a family design and highlight preferred family design alternatives.

Keywords: end-of-life management; product family design; component sharing

1. Introduction

For more than a decade, a great deal of research has been conducted on the design issues expressed in the following questions. Can a set of products benefit a company when it is designed to have common components? If so, what are the best designs for a group of products? It is commonly accepted now that sharing components across multiple products can have a multitude of benefits, especially in the design and manufacturing stages. Specifically, component sharing is highlighted as a means of increasing product variety while retaining the necessary economies of scale and scope (Simpson *et al.* 2006). The growing interest in component sharing has triggered the development of product family design. Many approaches have been developed to support component sharing and product family design, and successful product families have been reported by both academics and industries.

Most existing methods and applications, however, have overlooked the impact of product family design on end-of-life management. Managing end-of-life products involves two major activities; *i.e.* product take-back for collecting used products from their former users and end-of-life recovery

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ISSN 0305-215X print/ISSN 1029-0273 online

DOI: 10.1080/0305215X.2010.482990
http://www.informaworld.com

of economic value. Environmental regulations currently mandate that manufacturers assume the economic burden of these two activities (Mangun and Thurston 2002); therefore, manufacturers must find a way to achieve profitability in end-of-life management. The point is that the profitability of end-of-life management may be influenced by the design of the product family.

End-of-life management involves multiple types of end-of-life products. Accordingly, product take-back and end-of-life recovery are influenced by individual product designs and the interactions between designs; *i.e.* the commonality of components across product variants. Manufacturers must carefully make commonality decisions in product family design to improve profitability of end-of-life management. Thus, a method is needed to determine which product family design is better from an end-of-life perspective.

This article presents a quantitative model for assessing the profitability of product family designs in end-of-life management. The proposed model evaluates a product family for which the product variants are assumed to overlap end-of-life stages. Each product variant has a hierarchical assembly structure, and some of its components can be shared with other product variants. The model also focuses on the fact that component commonality influences the end-of-life profitability by increasing the degree of component interchangeability and identifies an optimal strategy for maximizing the profitability of managing product take-back and end-of-life recovery, which is formulated as a mixed integer programming problem.

Most previous product family design research has not focused on end-of-life management. Although a few studies (*e.g.* Simpson 1998, Perera *et al.* 1999, Bras 2007) considered the end-of-life stage, they simply state that cost reduction in the end-of-life stage is another possible advantage of component sharing. Our model quantitatively assesses the effects of component sharing on the end-of-life stage. The authors believe this is a novel approach in the product family and the end-of-life management area.

The rest of the article is organized as follows. The background for the article is presented in Section 2 and the problem settings and the end-of-life management process are described in Section 3, while mathematical model to assess product family design is proposed in Section 4, and an illustrative example is presented in Section 5. The conclusion of this study and future work are presented in Section 6.

2. Background

2.1. *End-of-life management of a family of products*

A family of products can be defined as a group of related products that share a product platform; *i.e.* a set of common design elements, processes, technologies, and other assets (Jiao *et al.* 2007, Simpson 2004). In this article, a product family is specifically defined as a group of products, (1) that has common components shared by some or all of its product variants, and (2) whose product variants are anticipated to have overlapping end-of-life stages; *i.e.* end-of-life management can be performed on multiple product variants simultaneously. Sharing the product platform can benefit both design (pre-life) and recovery (end-of-life) stages with this definition.

Figure 1 depicts an exemplary family of products in which two variants exist and Component X is common. Each product variant has a hierarchical assembly structure consisting of three levels; *i.e.* core, intermediate (Inderfurth and Langella 2008) and component. A core refers to a used product that is intact. Disassembly separates a core into parts that are either intermediates or components. Here, the term ‘part’ refers to any decomposable element of a product. Intermediate denotes non-atomic parts of a product at the middle level of product hierarchy, which are neither a core nor a component. Through another step of disassembly, intermediates can be separated into child components. Component indicates an atomic part at the lowest level, which cannot be disassembled any further (Krikke *et al.* 1998). The parent items of a component can be either

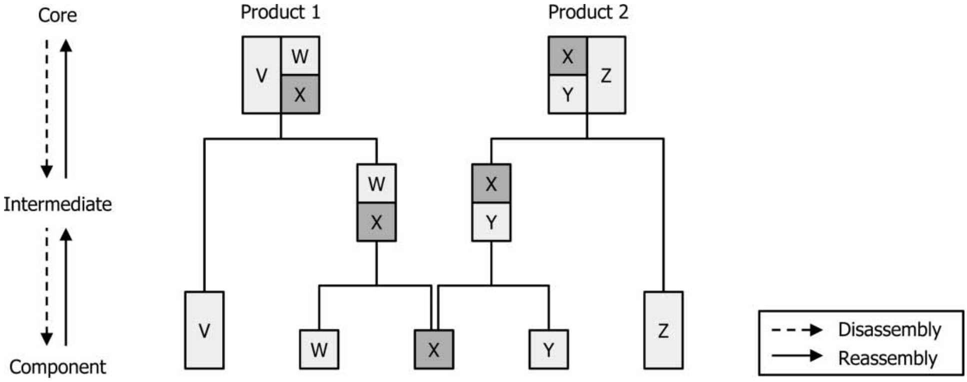

Figure 1. Exemplary product family sharing Component X.

intermediates or cores, depending on the product structure. Starting from components, child parts are reassembled into a parent part until a core is made. It should be noted that all product variants in this article are assumed to have three-level structures for simplicity.

With the definition of product family in the beginning of this section, product variants of a family of products have overlapping end-of-life stages. Hence, within a time period, multiple product variants are expected to reach the end-of-life stage at the same time, which renders component commonality across the variants affects profitability in end-of-life management. As Simpson (1998), Perera *et al.* (1999), and Bras (2007) stated, improving component commonality can benefit end-of-life management in two ways. First, the economies of scale in the recovery operation increase. Necessary tools and worker skill and set-up time decrease in various operations, including disassembly, conditioning, and reassembly. Second, the interchangeability of components across a family of products increases. For instance, in Figure 1, Component X, which resulted from the disassembly of Product 1, can be used for refurbishing both Intermediate WX and Product 1 and Intermediate XY and Product 2. Such increased interchangeability facilitates the profitable reuse of more components.

This article focuses on the increased interchangeability of components and its impact on manufacturer’s profit, which has not been dealt with to any great extent in the previous literature. When a product family has some components that are shared by multiple variants, this article proposes a quantitative model for determining how the interchangeable components of a product family can benefit end-of-life management.

2.2. *Relevant research*

In this research, end-of-life management is formulated as a problem of multiple recovered cores with commonalities. The background to this research is related to two research areas, *i.e.* end-of-life recovery management and disassembly-to-order.

2.2.1. *End-of-life recovery management*

The studies in this field aim at identifying optimal disassembly and reprocessing plans that maximize the recovery profit from returned cores. Main concerns include the optimal disassembly level and sequence as well as end-of-life options for resultant parts. Only the literature that considers both disassembly and recovery simultaneously is discussed here.

Early works in this field have focused on managing the end-of-life recovery of a single type of product. Penev and De Ron (1996) and Pnueli and Zussman (1997) converted a product structure into the form of an AND/OR graph and suggested algorithms to find optimal disassembly and recovery plans of the product. Building on the work of Lambert (2002), who developed a linear programming model to find the optimal disassembly sequence using a transition matrix, Kwak *et al.* (2009) proposed a model to simultaneously optimize both the disassembly sequence and end-of-life options. Some methods, such as those developed by Krikke *et al.* (1998) and Gonzalez and Adenso-Diaz (2005), have focused on optimizing the disassembly level (*i.e.* the extent to which a product is disassembled) rather than the disassembly sequence.

Several studies have considered an extension problem of multiple types of products. Jayaraman (2006) and Franke *et al.* (2006) developed methods to manage a number of units of multiple cores in end-of-life recovery. These models incorporated refurbishment in the optimization model at an abstract level. Unlike other studies focusing on component commonality, Behdad *et al.* (2009) elucidated process commonality across multiple products in end-of-life recovery. A recovery management model for multiple products was developed, assuming the existence of common disassembly operations. The model by Behdad *et al.* is applicable to multiple products that do not share any components.

To incorporate component sharing and its impact on component interchangeability, a model should be able to simultaneously consider the end-of-life management of multiple products and the reuse of disassembled parts in the refurbishment process. Existing methods do not meet these requirements. Kwak and Kim (2009) dealt with the refurbishment option at a process level, creating the opportunity for in-house component reuse for refurbishment, yet their model is focused on a single-type product only.

2.2.2. *Disassembly-to-order*

The research in this arena addresses the problem of scheduling disassembly. The demand for parts or recovered products triggers disassembly, and the objective is to fulfil the demand at minimum cost. In general, multiple types of cores are assumed, and deterministic demand for recovered items is given at the beginning. Key decision variables are the amount and type of cores to acquire and disassemble and the amount and type of parts to externally procure.

Taleb and Gupta (1997) proposed the problem of scheduling disassembly for multiple products with component or material commonality. They presented two algorithms to make two decisions, *i.e.* the quantity of cores to buy and the operation schedule for disassembly. Meacham *et al.* (1999) formulated a single-period optimization model to determine the cost-minimizing disassembly plan for multiple products. Ferrer and Whybark (2001) extended previous methods by incorporating multiple factors (*i.e.* multiple periods, core trade-ins, disassembly yield) and the model gives an optimal plan that minimizes total inventory costs while satisfying the demand for parts. Imtanavanich and Gupta (2004, 2005) developed a multi-criteria decision making model that incorporates product deterioration, stochastic yields, and demand for parts from material recycling. Inderfurth and Langella (2008) introduced linear programming models that were more generalized than previous models. They considered multi-level product structures and partial disassembly in the optimization model.

These studies provide an excellent background for this research. However, the objective of the studies in the area above was to fill orders with minimum costs, not to maximize the profit from end-of-life management. In addition, these studies concentrated more on the disassembly schedule than the recovery option. The proposed model is a new contribution that is distinct in three ways:

(1) Instead of assuming a given number of cores to recover, the model decides how many cores and which types and what conditions of cores should be collected to maximize the end-of-life management profit.
(2) In formulating end-of-life management as a problem of multiple cores with commonality, the proposed model incorporates product refurbishment at a detailed level.

(3) The core collection target and the rate of recovery are considered as constraints to comply with environmental regulations.

This new set of contributions is described in the following sections.

3. Processes for end-of-life management

3.1. *Product take-back*

End-of-life management consists of two sequential processes, *i.e.* product take-back and end-of-life recovery. Product take-back is the process of collecting cores, *i.e.* products that reach their end-of-life status. Since product take-back determines the volume, type, and quality of feedstock processed later in the recovery process, how many cores and which types of cores should be acquired are major concerns for the manufacturer.

Regulatory requirements on waste collection greatly affect manufacturers' take-back decisions by forcing manufacturers to meet a certain collection target. For example, the Waste Electrical and Electronic Equipment (WEEE) directive imposes a mandatory collection target of four kilograms per person per year on EU member states. Recently, the directive announced a new target, 65% of the average weight of products positioned on the market over the two previous years in each member state. In this article, the proposed model assumes that a collection target exists for a manufacturing company to comply with the legislation. The company must take back a certain number of cores so that the total weight of the collected cores exceeds the target.

The cost of core procurement is another important factor that affects take-back decisions. According to environmental legislation, consumers can return the cores to collection points free of charge in most cases. Without compensation, however, consumers tend to store a core indefinitely even if they no longer use it. Manufacturers provide an economic incentive to motivate consumers to return their cores. Although this may increase the take-back cost, manufacturers can secure a greater number of valuable cores in order to offset end-of-life management costs by making more profit in recovery. Thus, the proposed model assumes a buy-back programme as a take-back strategy. The buy-back price can have either negative, zero, or positive value depending on the type and condition of the core. Negative value is included, because a company is allowed to charge consumers for taking back cores in some cases (Envirowise, 2004).

For simplicity, this article adopts bi-level condition levels, *i.e.* fully-functioning (referred to as 'working' hereafter) and malfunctioning (referred to as 'non-working' hereafter). Working cores are usually more expensive to buy back but have higher disassembly yield rates of working parts and components. Hence, the type, condition, and number of cores to take back should be carefully determined in end-of-life management.

3.2. *End-of-life recovery*

After product take-back, the collected cores pass thorough an end-of-life recovery process. Manufacturers must identify the most profitable way to recover incoming feedstock. To this end, this research considers recycling, reuse, reconditioning, refurbishment, and cannibalization as recovery options (Krikke *et al.* 1998, Jacobsson 2000, Kwak and Kim 2009). The meaning of each option is described in Table 1. Here, reuse and reconditioning options only apply to working items.

When deciding how to recover cores, manufacturers also need to consider environmental regulations, which obligate them to achieve a specific recovery rate or pay a penalty. In the proposed model, the recovery target is set at 80% of the collection target. For example, a company that has a collection target of 85,000 lb should recover more than 68,000 lb of resources from the collected

Table 1. End-of-life recovery options.

Option	Description
Recycling	An item is sent to recyclers and shredded, separated, and refined to recover raw materials. The higher per weight material concentration, the more per weight recycling revenue.
Reuse	An item is sold to another user to be used for its original purpose. Only essential operations (*e.g.* data scrubbing) are conducted without any value-adding operations. Only working cores can follow this option.
Reconditioning	An item is sold to another user and used for its original purpose. In addition to essential operations, some minor value-adding operations, such as cleaning, lubricating, and polishing, are conducted to raise the value of the core. Only working cores can follow this option.
Refurbishment	An item is restored to its original condition. Product type and structure are maintained. Disassembly, part conditioning and replacement, and reassembly belong to the refurbishment option. If upgrading functions are conducted to the level of up-to-date products, such refurbishment can be reclassified as remanufacturing.
Cannibalization	An item is cannibalized for parts. Disassembly is conducted to separate a core into a set of parts. Individual parts resulting from the disassembly then can start their recovery as independent units, each with its own recovery and disposal option. Working parts can also be a source of parts for refurbishing other parts or cores.

cores. In other words, the firm cannot dispose of more than 17,000 lb. Many manufacturers (*i.e.* HP, Dell and Apple) prefer to satisfy the regulation rather than to pay penalties for promoting 'green' corporate image. Therefore, the proposed model represents the regulation as a constraint, which must be satisfied.

Figure 2 depicts the three-stage recovery process considered in this research. Here, a company is assumed not to carry out recycling operations on its own account. Instead, the company sells cores and parts to its recycling partners who perform actual recycling operations. Depending on the path each core follows in the recovery process, a set of collected cores can be transformed into eight kinds of outputs, *i.e.* four in the form of a product and four in the form of parts. These outputs are further transported to landfills, recycling partners, or customer markets, according to their assigned disposal and recovery options.

In the first stage of the recovery process, a decision is made concerning the next step for each core collected from the product take-back. To illustrate, suppose a set of used cell phones just arrived for the recovery process. Based on their conditions, the cell phones are discarded, recycled, reused, reconditioned, or disassembled. Cell phones for disposal and material recycling go to landfills or to recycling partners. The other cell phones undergo data scrubbing to eliminate any remaining personal data, and some of them are resold as used or reconditioned phones, and some are sent to Stage 2 for disassembly.

In Stage 2, a core is disassembled for the purpose of refurbishment or cannibalization. For example, a cell phone from Stage 1 is disassembled into a screen module, main board, antenna, microphone, keypad, and cases. Further disassembly can be done as needed, but an important point is that every resultant part is either working or non-working. A deterministic parameter, disassembly yield rate, reflects the number of working parts acquired by the disassembly of a core or an intermediate. Similar to the approach taken by Krikke *et al.* (1998), disassembly yield rate in this research depends on the parent item's condition. For example, suppose a cell phone has the following disassembly yield rates for its main board: $\text{Yield}\,|\text{W} = 1$ and $\text{Yield}\,|\text{N} = 0.8$. When a working (W) cell phone is disassembled, one unit of working main board results from the disassembly. When a non-working (N) cell phone is disassembled, only 0.8 unit of working mother board is harvested. The remaining 0.2 unit is non-working.

After disassembly, non-working parts are either disposed of or recycled. For working parts, any disposal and recovery options are allowed including reassembly. If the reassembly option is chosen, a part is harvested, reconditioned, and sent to Stage 3. In Stage 3, parts are reassembled

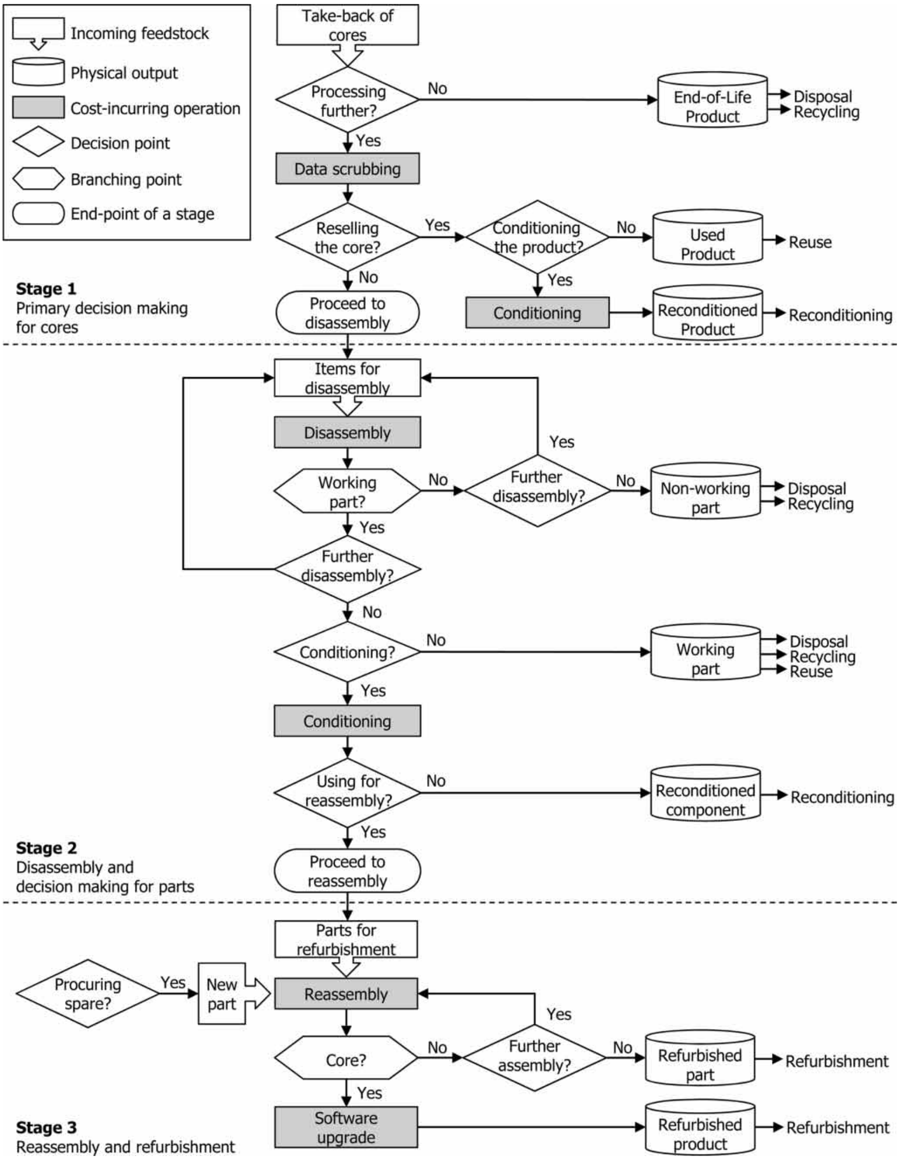

Figure 2. End-of-life recovery process.

into its parent part or a core. When there is a shortage of parts, new spare parts are procured. The resulting parts and cores are remarketed as refurbished items.

4. Model for assessing product family design from an end-of-life perspective

4.1. *Problem statement*

This article proposes a model for assessing product family design from the end-of-life perspective, focusing on component interchangeability. Given a product family design, the proposed model uses mixed integer programming to identify an optimal take-back and recovery strategy. The optimization result can be used to quantify the economic impacts of component sharing on end-of-life management. The proposed model is summarized by the following optimization problem:

(1) Given
  - Product family design in which commonality decisions are already identified
  - Disassembly yield rates of cores and intermediates
  - Costs of cores and the maximum amount of cores available for take-back
  - Costs and revenue of executing recovery and disposal options
  - Market demand for recovered items

(2) Find
  - Optimal take-back strategy: Amount, type, and condition of core that should be taken back
  - Optimal disposal and recovery strategy: Amount, type, and condition of core that should follow each disposal and recovery option; disassembly level of a core (parts to which a core should be disassembled) and recovery and disposal options for parts; amount and type of spare parts to acquire for refurbishment.

(3) Subject to
  - Flow volume balance constraints: With respect to an item, its flow balance between input and output units should be maintained.
  - Environmental regulations: Collection and recovery targets should be satisfied.
  - Core availability: There are limits on the amount of available cores that can be collected.
  - Avoiding excess fulfilment: The supply of a recovered item cannot exceed the demand for it.

(4) Maximizing
  - Total net profit from managing the end-of-life of a family of products.

(5) Assuming
  - Three-level product structure: Each product variant has a three-level assembly structure consisting of a core, intermediates, and components, which are denoted with three indices.
  - Unlimited part procurement: Spare parts can be procured with no lead time, and there are no limits on the number of parts that can be purchased.
  - Unlimited facility capacity: There are no limits on the number of items or the number of operations that can be processed.
  - No loss in yield in the recovery operation: Data scrubbing, conditioning, disassembly, and reassembly do not damage their input items, and there is no loss in yield caused by operations.
  - Deterministic parameter values: Disassembly yield rates, market demand, related costs, and revenue are deterministic.
  - Single-period planning.

4.2. *Mathematical formulation*

4.2.1. *Objective function*

The objective of this model is to maximize the total profit from end-of-life management. The objective function is modelled in Equation (1). The total cost of end-of-life management is the sum of nine cost components:

(1) Take-back (C_1).
(2) Data scrubbing (C_2).
(3) Product conditioning (C_3).
(4) Disassembly (C_4).

(5) Part conditioning (C_5).
(6) Spare part procurement (C_6).
(7) Reassembly (C_7).
(8) Software update (C_8).
(9) Disposal (C_9).

The total recovery revenue is the sum of four revenue terms: revenue from selling items to recyclers (R_1), revenue from selling used items to the market (R_2), revenue from selling reconditioned items to the market (R_3) and revenue from selling refurbished items to the market (R_4). The notations used in this article are described in Table 2.

$$min: \quad \sum_{n=1}^{9} C_n - \sum_{n=1}^{4} R_n \tag{1}$$

$$where \quad C_1 = \sum_{i \in I} (c^t_{i,w} \cdot X^t_{i,w} + c^t_{i,n} \cdot X^t_{i,n}) \tag{2}$$

$$C_2 = \sum_{i \in I} c^e_i \cdot (X^u_{i,w} + X^c_{i,w} + X^d_{i,w} + X^d_{i,n}) \tag{3}$$

$$C_3 = \sum_{i \in I} c^c_i \cdot X^c_{i,w} \tag{4}$$

$$C_4 = \sum_{i \in I} c^d_i \cdot (X^d_{i,w} + X^d_{i,n}) + \sum_{j \in J} c^d_j \cdot (X^d_{j,w} + X^d_{j,n}) \tag{5}$$

$$C_5 = \sum_{j \in J} c^c_j \cdot (X^c_{j,w} + X^r_{j,w}) + \sum_{k \in K} c^c_k \cdot (X^c_{k,w} + X^r_{k,w}) \tag{6}$$

$$C_6 = \sum_{j \in J} c^y_j \cdot Y_j + \sum_{k \in K} c^y_k \cdot Y_k \tag{7}$$

$$C_7 = \sum_{i \in I} c^r_i \cdot Z^s_i + \sum_{j \in J} c^r_j \cdot (Z^r_j + Z^s_j) \tag{8}$$

$$C_8 = \sum_{i \in I} c^s_i \cdot Z^s_i \tag{9}$$

$$C_9 = \sum_{i \in I} c^l_i \cdot (X^l_{i,w} + X^l_{i,n}) + \sum_{j \in J} c^l_j \cdot (X^l_{j,w} + X^l_{j,n}) + \sum_{k \in K} c^l_k \cdot (X^l_{k,w} + X^l_{k,n}) \tag{10}$$

$$R_1 = \sum_{i \in I} r^m_i \cdot (X^m_{i,w} + X^m_{i,n}) + \sum_{j \in J} r^m_j \cdot (X^m_{j,w} + X^m_{j,n}) + \sum_{k \in K} r^m_k \cdot (X^m_{k,w} + X^m_{k,n}) \tag{11}$$

$$R_2 = \sum_{i \in I} r^u_i \cdot X^u_{i,w} + \sum_{j \in J} r^u_j \cdot X^u_{j,w} + \sum_{k \in K} r^u_k \cdot X^u_{k,w} \tag{12}$$

$$R_3 = \sum_{i \in I} r^c_i \cdot X^c_{i,w} + \sum_{j \in J} r^c_j \cdot X^c_{j,w} + \sum_{k \in K} r^c_k \cdot X^c_{k,w} \tag{13}$$

$$R_4 = \sum_{i \in I} r^z_i \cdot Z^s_i + \sum_{j \in J} r^z_j \cdot Z^s_j \tag{14}$$

Table 2. Mathematical notation.

Notation	Description
Index	
I	Core set; $i \in I$
J	Intermediate set; $j \in J$
K	Component set; $k \in K$
P_j, P_k	Parent set of intermediate j and parent set of component k, respectively
Q	Quality condition set; $Q = \{w, n\}$; $q \in Q$
w, n	Working and non-working quality condition index, respectively
Variable	
$X^t_{i,q}$	Number of core i with condition q to take back
$X^l_{i,q}$, $X^l_{j,q}$, $X^l_{k,q}$	Number of core i, intermediate j, and component k with condition q to dispose of, respectively
$X^m_{i,q}$, $X^m_{j,q}$, $X^m_{k,q}$	Number of core i, intermediate j, and component k with condition q to recycle, respectively
$X^u_{i,q}$, $X^u_{j,q}$, $X^u_{k,q}$	Number of core i, intermediate j, and component k with condition q to reuse, respectively
$X^c_{i,q}$, $X^c_{j,q}$, $X^c_{k,q}$	Number of core i, intermediate j, and component k with condition q to recondition, respectively
$X^d_{i,q}$, $X^d_{j,q}$	Number of core i and intermediate$j$ with condition q to disassemble, respectively
$X^r_{j,q}$, $X^r_{k,q}$	Number of intermediate j and component k with condition q to use in refurbishment, respectively
Y_j, Y_k	Number of intermediate j and component k to procure for spare, respectively
Z^r_j	Number of intermediate j to refurbish and use in core refurbishment
Z^s_i, Z^s_j	Number of core i and intermediate j to refurbish and sell in the market
Parameter	
$\pi^o_{i,j}$	Number of units of intermediate j originally included in core i; the multiplicity of intermediate j
$\pi^o_{i,k}$	Number of units of component k originally included in core i; the multiplicity of component k
$\pi^o_{j,k}$	Number of units of component k originally included in intermediate j; the multiplicity of component k
$\pi^q_{i,j}$	Disassembly yield rates of core i with condition q with respect to working intermediate j
$\pi^q_{i,k}$	Disassembly yield rates of core i with condition q with respect to working component k
$\pi^q_{j,k}$	Disassembly yield rates of intermediate j with condition q with respect to working component k
α, β	Collection target and the maximum allowed disposal amount (recovery target)
$A_{i,q}$	Number of core i with condition q available for take-back
ω_i, ω_j, ω_k	Weight of core i, intermediate j, and component k, respectively
$c^t_{i,q}$	Per unit take-back cost for core i with condition q
c^e_i	Per unit data scrubbing cost for core i
c^c_i, c^c_j, c^c_k	Per unit conditioning cost for core i, intermediate j, and component k, respectively
c^d_i, c^d_j	Per unit disassembly cost for core i and intermediate j, respectively
c^r_i, c^r_j	Per unit reassembly cost for core i and intermediate j, respectively
c^y_j, c^y_k	Per unit procurement cost for intermediate j and component k, respectively
c^s_i	Per unit software upgrade cost for core i
c^l_i, c^l_j, c^l_k	Per unit cost from disposing of core i, intermediate j, and component k, respectively
r^m_i, r^m_j, r^m_k	Per unit revenue from recycling core i, intermediate j, and component k, respectively
r^u_i, r^u_j, r^u_k	Per unit revenue from reusing core i, intermediate j, and component k, respectively
r^c_i, r^c_j, r^c_k	Per unit revenue from reconditioning core i, intermediate j, and component k, respectively
r^z_i, r^z_j	Per unit revenue from refurbishing core i and intermediate j, respectively
D^u_i, D^u_j, D^u_k	Demand for used core i, intermediate j, and component k, respectively
D^c_i, D^c_j, D^c_k	Demand for reconditioned core i, intermediate j, and component k, respectively
D^z_i, D^z_j	Demand for refurbished core i, and intermediate j, respectively

4.2.2. *Constraints*

Flow balance of cores. There are several ways to process collected working cores, *i.e.* sending to landfills, selling to recyclers, selling as a used product, selling as a reconditioned product, and conducting disassembly to refurbish or cannibalize the core. For non-working cores, the three available options are disposal, recycling, and disassembly. Constraints (15) and (16) require every collected core to follow one of the possible options.

$$X^t_{i,w} = X^l_{i,w} + X^m_{i,w} + X^u_{i,w} + X^c_{i,w} + X^d_{i,w} \quad \forall i \in I \tag{15}$$

$$X^t_{i,n} = X^l_{i,n} + X^m_{i,n} + X^d_{i,n} \quad \forall i \in I \tag{16}$$

Flow balance of intermediates. Constraints (17) and (18) restrain the flow balance of working and non-working intermediates, respectively. The left-hand side of each constraint represents the amount of intermediates obtained from the disassembly of their parent cores. Since the model assumes a bi-level quality condition for every item, every intermediate acquired from the disassembly is either working or non-working. Depending on the condition of the parent cores, the amount of working and non-working intermediates can vary. To reflect this, the number of disassembled working cores and non-working cores are multiplied by different disassembly yields π.

$$\sum_{i \in P_j} (\pi^w_{i,j} \cdot X^d_{i,w} + \pi^n_{i,j} \cdot X^d_{i,n}) = X^l_{j,w} + X^m_{j,w} + X^u_{j,w} + X^c_{j,w} + X^d_{j,w} + X^r_{j,w} \quad \forall j \in J \tag{17}$$

$$\sum_{i \in P_j} ((\pi^o_{i,j} - \pi^w_{i,j}) \cdot X^d_{i,w} + (\pi^o_{i,j} - \pi^n_{i,j}) \cdot X^d_{i,n}) = X^l_{j,n} + X^m_{j,n} + X^d_{j,n} \quad \forall j \in J \tag{18}$$

Each earned intermediate must follow one of the possible processing options. For working intermediates, six options are available: disposal, recycling, reuse, reconditioning, disassembly into components, and reuse for core refurbishment. For non-working intermediates, only three options are available: disposal, recycling, and disassembly into components.

Flow balance of components. Constraints (19) and (20) ensure the flow balance of working and non-working components, respectively. The left-hand side of each constraint represents the amount of components that resulted from disassembly. Both a core and an intermediate can be the parents of a component depending on the product family design. The conditions of parent items determine the amount of working and non-working components.

Constraint (19) states that every working component must follow one of five options: disposal, recycling, reuse, reconditioning, and reuse for intermediate refurbishment. Constraint (20) requires every non-working component to be land-filled or recycled. Since a component is the lowest-level part, the option of disassembly is not considered in both constraints.

$$\begin{aligned} &\sum_{i \in P_k} (\pi^w_{i,k} \cdot X^d_{i,w} + \pi^n_{i,k} \cdot X^d_{i,n}) + \sum_{j \in P_k} (\pi^w_{j,k} \cdot X^d_{j,w} + \pi^n_{j,k} \cdot X^d_{j,n}) \\ &\quad = X^l_{k.w} + X^m_{k.w} + X^u_{k.w} + X^c_{k.w} + X^r_{k.w} \quad \forall k \in K \end{aligned} \tag{19}$$

$$\begin{aligned} &\sum_{i \in P_k} ((\pi^o_{i,k} - \pi^w_{i,k}) \cdot X^d_{i,w} + (\pi^o_{i,k} - \pi^n_{i,k}) \cdot X^d_{i,n}) + \sum_{j \in P_k} ((\pi^o_{j,k} - \pi^w_{j,k}) \cdot X^d_{j,w} \\ &\qquad + (\pi^o_{j,k} - \pi^n_{j,k}) \cdot X^d_{j,n}) = X^l_{k,n} + X^m_{k,n} \quad \forall k \in K \end{aligned} \tag{20}$$

Flow balance of refurbished intermediates. Intermediates can be refurbished by reassembling working components. Working components can result from the disassembly of cores or from external procurement. Once refurbished, intermediates can be sold on the market or reassembled with other parts to refurbish cores. Constraint (21) forces a balance of the flow between input components and output refurbished intermediates.

$$\sum_{j \in P_k} \pi^o_{j,k} \cdot (Z^r_j + Z^s_j) = X^r_{k,w} + Y_k \quad \forall k \in K \tag{21}$$

Flow balance of refurbished cores. Similar to intermediates, cores can be refurbished by reassembling their working child parts. Working intermediates can be obtained by core disassembly or intermediate refurbishment. If there is a shortage of working intermediates, external procurement is also possible. As for the working components, only two sources are available: core disassembly and external procurement. After reassembly, refurbished cores are sold in the market as refurbished products. Constraint (22) restricts the flow balance between input intermediates and output refurbished cores, while Constraint (23) balances the flow between input components and output refurbished cores.

$$\sum_{i \in P_j} \pi^o_{i,j} \cdot Z^s_i = X^r_{j,w} + Y_j + Z^r_j \quad \forall j \in J \tag{22}$$

$$\sum_{i \in P_k} \pi^o_{i,k} \cdot Z^s_i = X^r_{k,w} + Y_k \quad \forall k \in K \tag{23}$$

Environmental regulations. Environmental regulations require weight-based calculations. Constraint (24) represents the regulation on collection targets. The proposed model presumes a collection target α for a manufacturing company. The company must take back enough cores to exceed the target. Constraint (25) models the regulation on the minimum rate of recovery (or, the maximum allowable disposal amount). The left-hand side of the constraint represents the total weight of discarded items, and β denotes the upper limit of disposal. In the proposed model, the recovery target is set at 80% of the collection target α. In other words, disposal of up to 20% of α is allowed; thus, $\beta = 0.2\alpha$.

$$\sum_{i \in I} \omega_i \cdot (X^t_{i,w} + X^t_{i,n}) \geqslant \alpha \tag{24}$$

$$\sum_{i \in I} \omega_i \cdot (X^l_{i,w} + X^l_{i,n}) + \sum_{j \in J} \omega_j \cdot (X^l_{j,w} + X^l_{j,n}) + \sum_{k \in K} \omega_k \cdot (X^l_{k,w} + X^l_{k,n}) \leqslant \beta \tag{25}$$

Core availability. The proposed model assumes a buy-back programme wherein the manufacturer pays the consumer for each core. The number and type of cores to take-back are decision variables, not given parameters. Regarding take-back decisions, Constraint (26) limits the amount of available cores that can be collected.

$$X^t_{i,w} \leqslant A_{i,w};\ X^t_{i,n} \leqslant A_{i,n} \quad \forall i \in I \tag{26}$$

Demand satisfaction and avoidance of excess fulfilment. The customer market demands a certain amount of used, conditioned, and refurbished items. Constraint (27) prevents the supply of recovered cores from exceeding the market demand. Similarly, the supply of recovered intermediates and components cannot exceed the corresponding demand according to Constraints (28) and (29), respectively.

$$X^u_{i,w} \leqslant D^u_i;\ X^c_{i,w} \leqslant D^c_i;\ Z^s_i \leqslant D^z_i \quad \forall i \in I \tag{27}$$

$$X^u_{j,w} \leqslant D^u_j;\ X^c_{j,w} \leqslant D^c_j;\ Z^s_j \leqslant D^z_j \quad \forall j \in J \tag{28}$$

$$X^u_{k,w} \leqslant D^u_k;\ X^c_{k,w} \leqslant D^c_k \quad \forall k \in K \tag{29}$$

Variable condition. All decision variables in the model represent numbers of items. Due to disassembly yields, the amount of intermediates and components acquired from the disassembly might not be integers. To absorb the decimals, the amount of items sent to landfills and the amount sold to recyclers are set as real numbers. The others are constrained as integers. Constraints (30), (31), and (32) restrain these variable conditions.

$$X^t_{i,q}, X^u_{i,q}, X^c_{i,q}, X^d_{i,q}, Z^s_i \geqslant 0 \text{ and integer};\ X^l_{i,q}, X^m_{i,q} \geqslant 0 \quad \forall i \in I, \forall q \in Q \tag{30}$$

$$X^u_{j,q}, X^c_{j,q}, X^d_{j,q}, X^r_{j,q}, Y_j, Z^r_j, Z^s_j \geqslant 0 \text{ and integer};\ X^l_{j,q}, X^m_{j,q} \geqslant 0 \quad \forall j \in J, \forall q \in Q \tag{31}$$

$$X^u_{k,q}, X^c_{k,q}, X^r_{k,q}, Y_k \geqslant 0 \text{ and integer};\ X^l_{k,q}, X^m_{k,q} \geqslant 0 \quad \forall k \in K, \forall q \in Q \tag{32}$$

5. Illustrative example

This section presents an illustrative example with a smartphone family to illustrate how to apply the proposed model and how it supports decision making in product family design.

5.1. *Smartphone family design*

Design scenario. Suppose that a smartphone manufacturer makes new products in the first period and uses cores to offer second-hand items along with new products in the next period. Until now, the company has customized the design of each phone to a specific market segment using uniquely designed components. However, since the company offers various types of phones to the market at the same time, parts proliferation due to the core variety (Bras 2007) has become one of the biggest obstacles to making profits in recovery. To address this issue, the company is considering designing a family of products in which some parts are shared by product variants. The design team has developed a design alternative for a product family. Now, they want to know whether the family design actually supports the recovery business and, if so, what increase in profit is anticipated. Regarding the legislative issues, the company currently has a collection target of 85,000 lb and a recovery target of 68,000 lb.

In this scenario, the proposed model is applied to a smartphone family (composed of four product variants) and to a reference case. In the reference case, no component is shared by multiple product variants. Figure 3 represents the product structure of the smartphone considered in this research. All product variants have identical structures composed of 15 components. The design difference comes from the variant parts, represented as an oval in the figure. Some, but not all, product variants can share the identical design for the variant parts. If all product variants share the same design for a part, the part is referred to as a common part (Thevenot and Simpson 2006). The smartphone variants differ in memory size and rear panel colour. Table 3 gives detailed information on the part composition of each product variant. Four product variants in this product family share a significant number of parts and intermediates as noted as ‘common’. Number ‘1’ in Table 3 represents each type. For example, camera is noted as ‘1’ for all four variants (*i.e.* common part), while rear casing is noted as 1, 2, 3, or 4, each representing different component.

Finally, for a simple illustration, the second-hand items to be recovered from cores are assumed to maintain their original design, without any hardware upgrade. In addition, the proposed model is applicable when the refurbished items have different design from cores; such refurbished items are regarded also as cores while their take back availability ($A_{i,q}$) is set as zero. By doing so, no

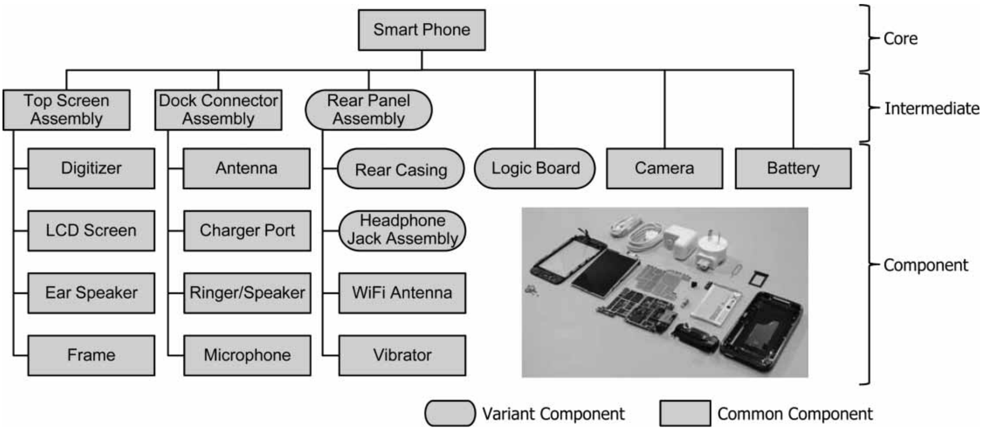

Figure 3. Smart phone structure (picture courtesy of iFixit; http://www.iFixit.com).

Table 3. Part composition of product variants in the high-sharing smartphone family.

Part	Type of Part	Type of Commonality	Phone 1 (8 GB, Black)	Phone 2 (16 GB, Black)	Phone 3 (16 GB, White)	Phone 4 (32 GB, Black)
Top screen assembly	Intermediate	Common	1	1	1	1
Dock connector assembly	Intermediate	Common	1	1	1	1
Rear panel assembly	Intermediate	Variant	1	2	3	4
Logic board	Component	Variant	1	2	2	3
Camera	Component	Common	1	1	1	1
Battery	Component	Common	1	1	1	1
Digitizer	Component	Common	1	1	1	1
LCD screen	Component	Common	1	1	1	1
Ear speaker	Component	Common	1	1	1	1
Frame	Component	Common	1	1	1	1
Antenna	Component	Common	1	1	1	1
Charger port	Component	Common	1	1	1	1
Ringer/speaker	Component	Common	1	1	1	1
Microphone	Component	Common	1	1	1	1
Rear casing	Component	Variant	1	2	3	4
Headphone jack assembly	Component	Variant	1	1	2	1
WiFi antenna	Component	Common	1	1	1	1
Vibrator	Component	Common	1	1	1	1

take back is considered for the second-generation products but they become a possible throughput from refurbishment.

5.2. *Parameter setting*

Table 4 represents the amount of available cores to take-back and the buy-back price of a core for each type and condition. The parameter values used here are simulated based on the actual prices of a particular smartphone in the new product market (www.apple.com), in the second-hand market (www. ebay.com), and in the buy-back market (www.gazelle.com; www. nextworth.com). The price difference between cores originates mostly from the difference in memory size, which is determined by the logic board, the most expensive component in the smartphone family.

Table 4. Product take-back information.

Type of core	Sale price ($) (without 2-year Contract)	Buy-back price ($)		Available units	
		Working	Non-working	Working	Non-working
Phone 1 (8 GB, Black)	450	150	100	80,000	100,000
Phone 2 (16 GB, Black)	550	180	100	50,000	80,000
Phone 3 (16 GB, White)	550	180	100	60,000	80,000
Phone 4 (32 GB, Black)	650	300	150	10,000	10,000

Table 5(a). Disassembly yield rate of cores.

Child part	Phone	
	Yield|W	Yield|N
Top screen assembly	1	0.333
Dock connector assembly	1	0.741
Rear panel assembly	1	0.600
Logic board	1	0.793
Camera	1	0.787
Battery	1	0.792

Table 5(b). Disassembly yield rate of intermediates.

Top screen assembly			Dock connector assembly			Rear panel assembly		
Child part	Yield|W	Yield|N	Child part	Yield|W	Yield|N	Child part	Yield|W	Yield|N
Digitizer	1	0.380	Antenna	1	0.587	Rear casing	1	0.407
LCD screen	1	0.545	Charger port	1	0.365	Headphone jack assembly	1	0.478
Ear speaker	1	0.718	Ringer/speaker	1	0.606	WiFi Antenna	1	0.496
Frame	1	0.804	Microphone	1	0.587	Vibrator	1	0.496

Table 5(a) shows the disassembly yield rates of cores and intermediates. In the smartphone family, most failures are expected in the top screen assemblies, especially the digitizers. The yield rates used in this study are estimated based on the failure reports on a particular smartphone model (SquareTrade 2008, 2009). The model has a structure similar to the one in Figure 3. It should be also pointed out that, for simplicity, this study assumes the same yield rates for every core in the family. This is so for every rear panel assembly. If the parameter values are given, the proposed model can serve other cases as well. For example, the model is applicable to the case where different cores (*i.e.* Phone 1 and Phone 2) or different intermediates (*i.e.* rear panel assembly 1 and rear panel assembly 2) have different yield rates.

Finally, recovery cost and revenue parameters are assigned as shown in the Appendix. The disposal cost and recycling revenue of an item are assigned based on its weight shown in the second column. A cost per pound multiplier, $0.02/lb (Sodhi and Reimer 2001), is used to estimate disposal costs. For recycling revenue, three different multipliers are used: $5.00/lb for logic boards, $1.50/lb for batteries, and $2.50/lb for any mix of items (www.grn.com). Revenue from selling reused, reconditioned, and refurbished core is set as 30%, 40%, and 50% of the new product price in Table 5(a). As for parts, the ratios change to 50%, 65%, and 80% of new part price in the market. Retail prices of new parts are estimated according to the prices of similar parts in the market (www. ubreakifix.com).

Table 6. Optimization result (objective value).

	Family 1 (high-sharing design)	Family 2 (sharing display only)	Family 3 (sharing MIC only)	Reference (no sharing)
Cost in total	50,193,370	50,413,117	50,454,510	50,411,329
Take-back	35,428,200	35,590,450	35,590,420	35,590,420
Data scrubbing	433,395	433,358	433,358	433,358
Core conditioning	36,804	30,000	30,000	30,000
Disassembly	516,793	519,440	519,976	519,441
Part conditioning	694,243	614,578	611,375	616,448
New part procurement	12,507,263	12,652,324	12,696,412	12,648,694
Reassembly	476,674	472,969	472,970	472,970
Software upgrade	100,000	100,000	100,000	100,000
Disposal	0	0	0	0
Revenue in total	70,494,557	68,944,233	68,843,196	68,788,606
Recycling	64,017	74,574	75,056	75,104
Reuse	12,580,000	12,788,185	12,686,640	12,632,003
Reconditioning	18,830,540	17,061,474	17,061,500	17,061,500
Refurbishment	39,020,000	39,020,000	39,020,000	39,020,000
Objective value (cost-revenue)	−20,301,186	−18,531,117	−18,388,687	−18,377,277
ROI (return on investment)	40.45%	36.76%	36.45%	36.45%

Table 7. Optimal number of core to take-back.

	Working core				Non-working core			
	Family 1	Family 2	Family 3	Reference	Family 1	Family 2	Family 3	Reference
Phone 1 (8 GB, Black)	20,000	19,999	20,000	20,000	65,548	64,700	64,701	64,701
Phone 2 (16 GB, Black)	20,000	20,000	19,999	19,999	72,103	72,103	72,101	72,101
Phone 3 (16 GB, White)	20,000	20,000	20,000	20,000	72,103	72,103	72,104	72,104
Phone 4 (32 GB, Black)	9176	10,000	10,000	10,000	10,000	10,000	10,000	10,000
Total sum of weight (lb)	20,373	20,620	20,620	20,620	64,627	64,380	64,380	64,380

5.3. *Optimization result*

In order to assess how much profit can be improved by adopting the family design, a reference case without component sharing was necessary. Therefore, a set of four smartphones that share no components (reference case in column 5 in Table 6) was analysed in addition to the high-sharing family design (Family 1) described in Table 3, using equivalent parameters and assumptions. In addition, two families of smartphones with limited sharing are also derived and compared to examine how the degree of sharing influences the optimization result. One family (Family 2) shares only the digitizer and LCD screen across all product variants and the other one (Family 3) shares the microphone only.

The optimization results from the four different cases are shown in Table 6. Table 7 shows the optimal amount of cores to take back in each case. Due to the limitation of space, a complete set of optimization results is presented only for the high-sharing design in Table 8. Figure 4 is presented to help in understanding the table. It graphically represents some of the results in Table 8, specifically the results related to all cores, the front screen assembly, and the components that compose the front screen assembly.

From the optimization results, three implications are obtained as follows:

- *Result 1:* Family 1 is the most profitable design among the four cases. In Table 6, all four cases present negative objective values, which implies that the end-of-life management can

Table 8. Optimal solution to the end-of-life management of a smart phone family.

Item	X^t_w	X^t_n	X^m_w	X^u_w	X^c_w	X^d_w	X^r_w	X^m_n	X^d_n	Y	Z^r	Z^s
Product 1	20,000.0	65,548.0	0.0	10,000	10,000	0	·	0.0	65,548	·	·	10,000
Product 2	20,000.0	72,103.0	0.0	10,000	10,000	0	·	0.0	72,103	·	·	20,000
Product 3	20,000.0	72,103.0	0.0	10,000	10,000	0	·	0.0	72,103	·	·	20,000
Product 4	9176.0	10,000.0	0.0	0	6804	2372	·	0.0	10,000	·	·	50,000
Front screen assembly	75,550.1	146,575.9	0.1	20,000	20,000	0	35,550	0.9	146,575	1	64,449	40,000
Lower dock assembly	165,209.7	56,916.3	0.7	20,000	20,000	26,591	98,618	0.3	56,916	1382	0	20,000
Real panel assembly 1	39,328.8	26,219.2	0.8	5000	5000	19,328	10,000	0.2	26,219	0	0	20,000
Real panel assembly 2	43,261.8	28,841.2	0.8	5000	5000	13,262	19,999	0.2	28,841	1	0	15,000
Real panel assembly 3	43,261.8	28,841.2	0.8	5000	5000	13,262	19,999	0.2	28,841	1	0	15,000
Real panel assembly 4	8,372.0	4000.0	0.0	0	5000	3372	0	0.0	4000	50,000	0	10,000
Logic board 1	51,979.6	13,568.4	31,979.6	5000	5000	·	10,000	13,568.4	·	0	·	·
Logic board 2	114,355.4	29,850.6	54,355.4	10,000	10,000	·	40,000	29,850.6	·	0	·	·
Logic board 3	10,302.0	2070.0	0.0	5000	5000	·	302	2070.0	·	49,698	·	·
Camera	175,318.4	46,807.6	35,318.4	20,000	20,000	·	100,000	46,807.6	·	0	·	·
Battery	176,417.2	45,708.8	36,417.2	20,000	20,000	·	100,000	45,708.8	·	0	·	·
Digitizer	55,698.5	90,876.5	0.5	20,000	20,000	·	15,698	90,876.5	·	88,751	·	·
LCD screen	79,883.4	66,691.6	0.4	20,000	20,000	·	39,883	66,691.6	·	64,566	·	·
Ear speaker	105,240.9	41,334.2	0.9	20,000	20,000	·	65,240	41,334.2	·	39,209	·	·
Frame	117,846.3	28,728.7	0.3	20,000	20,000	·	77,846	28,728.7	·	26,603	·	·
Antenna	60,000.7	23,506.3	0.7	20,000	20,000	·	20,000	23,506.3	·	0	·	·
Dock connector	47,365.3	36,141.7	0.3	20,000	20,000	·	7365	36,141.7	·	12,635	·	·
Loud speaker	61,082.1	22,424.9	1082.1	20,000	20,000	·	20,000	22,424.9	·	0	·	·
Microphone	60,000.7	23,506.3	0.7	20,000	20,000	·	20,000	23,506.3	·	0	·	·
Rear casing 1	29,999.1	15,547.9	0.1	5000	5000	·	19,999	15,547.9	·	1	·	·
Rear casing 2	25,000.3	17,102.7	0.3	5000	5000	·	15,000	17,102.7	·	0	·	·
Rear casing 3	25,000.3	17,102.7	0.3	5000	5000	·	15,000	17,102.7	·	0	·	·
Rear casing 4	5000.0	2372.0	0.0	0	5000	·	0	2372.0	·	10,000	·	·
Headphone jack assembly 1	64,192.7	30,829.3	0.7	15,000	15,000	·	34,192	30,829.3	·	10,808	·	·
Headphone jack assembly 2	27,048.0	15,055.0	2048.0	5000	5000	·	15,000	15,055.0	·	0	·	·
GPS antenna	92,822.9	44,302.1	0.9	20,000	20,000	·	52,822	44,302.1	·	7178	·	·
Vibrator	92,822.9	44,302.1	0.9	20,000	20,000	·	52,822	44,302.1	·	7178	·	·

Note: First two columns for parts indicate the number of working and non-working items obtained from parents' disassembly; all X^l_w and X^l_n are zero.

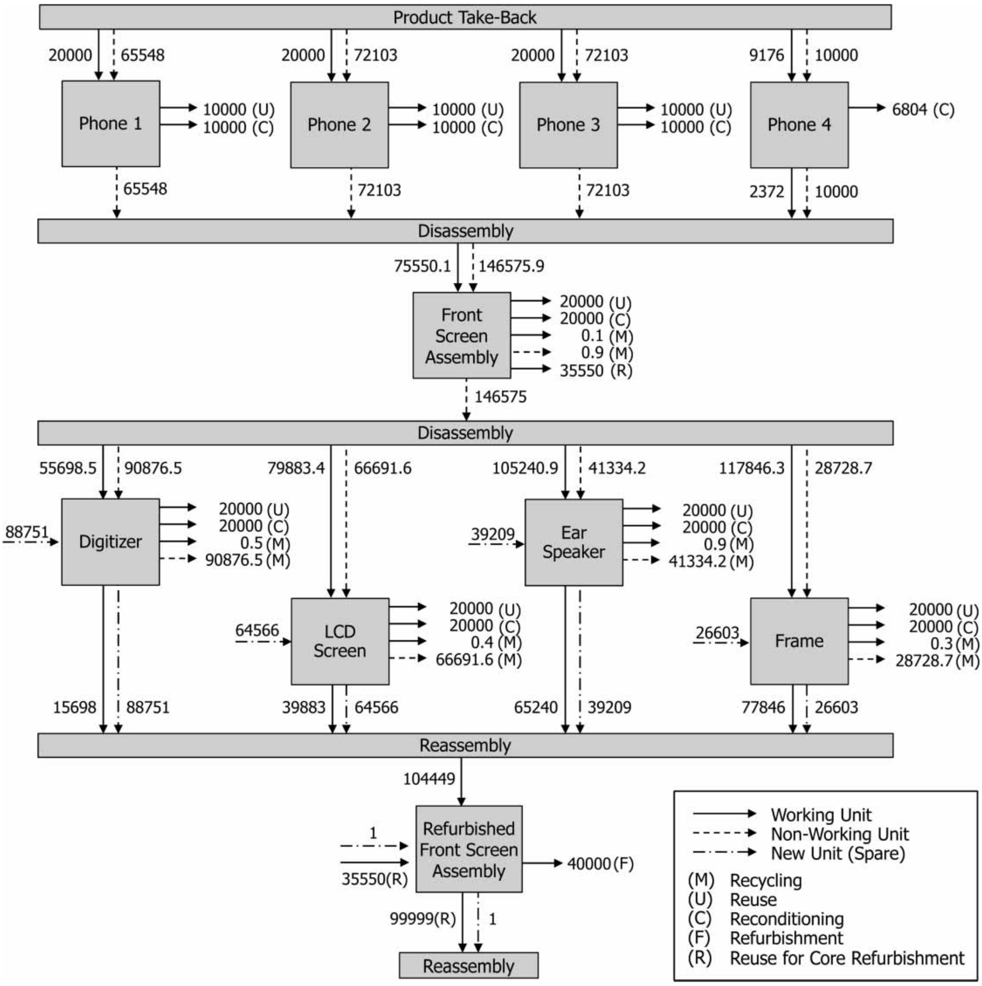

Figure 4. Graphical representation of the part of the optimal solution.

be a profitable business in all cases. Especially, Family 1 shows the smallest objective value (*i.e.* the largest profit) among the four cases. This means that Family 1 can support end-of-life management, and, once adopted, the profit is expected to increase by $1.9 million.

- *Result 2:* Family 1 allows the most efficient end-of-life management among the four cases. The last row of Table 6 presents the return on investment (ROI) of each case. ROI denotes the ratio of net profit relative to the cost, and the higher, the better. The ROI for Family 1 is the highest, which means that Family 1 can obtain more profit with the same investment.
- *Result 3:* Maximum profit and ROI increase as the degree of component sharing increases. Family 1 has the highest degree of component sharing, while the reference case has no sharing. Families 2 and 3 are in between these two. The four cases demonstrate that the maximum profit and ROI increase with the degree of component sharing. The results also indicate that the identity of the components that are shared is also an important factor affecting the profitability. For example, even though the microphone is shared in Family 3, the profit and ROI do not change much. Microphone and its parent intermediate (*i.e.* dock connector assembly) are the cheapest parts in a smartphone; although Family 3 encourages reuse of these parts (rather than material recovery), the revenue from the increased reuse is too small to make any significant

difference in total profit. However, when the digitizer and LCD screen are shared in Family 2, the profit increases more significantly. Family 2 facilitates reuse of digitizers and LCD screens, which are the most high-priced components along with logic boards.

The discussion up to now has been focused only on the economic perspective. Table 9 interprets the same optimization results from a different viewpoint, *i.e.* material flows. Comparing the four cases gives the following implications:

- *Result 4:* Family 1 requires less new resources to retrieve maximum profit from the same amount of input material. The table shows how much material must be input to the recovery system to obtain maximum profit. Family 1 shows superiority here as well. It uses a smaller amount of new resources. From an environmental perspective, less new material is usually more desirable. However, it is hard to conclude that the higher degree of sharing is always better in terms of saving resources. Families 2 and 3 require more weight of material than the reference case. (Also, Family 3 is worse than the reference, even in the net profit per unit weight.) This is due to higher reuse rate of parts. In other words, more reusable parts are available due to higher interchangeability in product family compared to the reference case. In turn, the re-manufacturer may use more material to manufacture more second-hand products for higher profit.
- *Result 5:* Family 1 supports end-of-life management to be more effective. First, Family 1 enables core management in a better way. From the environmental standpoint, reuse, reconditioning, and refurbishment are regarded as better options than material recycling. In this regard, Family 1 is superior to the others. Table 9 shows how the input material is processed in each design case. For Family 1, a greater percentage of material is reused, reconditioned, and refurbished than for the other cases. Second, Family 1 allows the retrieval of a greater value from the same amount of material. The last row of Table 9 shows net profit per pound. Family 1 shows the best performance in this area as well.

Family 1's superiority is the result of high interchangeability of its components. Thus, Family 1 can reduce take-back and part procurement costs while increasing recovery revenue. As shown in Table 4, Phone 4, which has 32 GB of memory and is black in colour, is the most profitable phone to refurbish. The unit net profit obtained from refurbishment is higher than it is for other variants. In addition, the current setting of parameters assumes a large market demand for a refurbished Phone 4. (The demand for every core and part is listed in the last column of Table A1 in the Appendix.) However, Product 4, the most preferred core to refurbish, is also the most difficult one to refurbish. Not only is it expensive to take-back, but the core availability is too low to satisfy the demand. While the demand for refurbished Phone 4 is 50,000, there are only 20,000 cores

Table 9. Material input-output flow.

		Family 1	Family 2	Family 3	Reference
Input	Take-back	85,000	85,000	85,000	85,000
	New part spare	13,128	16,529	16,545	16,454
Output	Disposal	0 (0.00%)	0 (0.00%)	0 (0.00%)	0 (0.00%)
	Recycling	21,482 (21.89%)	26,717 (26.31%)	26,910 (26.50%)	26,983 (26.58%)
	Reuse	17,988 (18.33%)	18,192 (17.92%)	18,016 (17.74%)	17,916 (17.65%)
	Reconditioning	20,554 (20.95%)	18,516 (18.24%)	18,516 (18.23%)	18,516 (18.24%)
	Refurbishment	38,104 (38.83%)	38,104 (37.53%)	38,104 (37.52%)	38,104 (37.53%)
Sum of weight (lb)		98,128	101,529	101,545	101,454
Net profit per pound		206.88	182.52	181.09	181.12

available, including working and non-working cores. Therefore, spare parts must be purchased to meet the demand.

In the case of Family 1, a company can utilize other phones to refurbish Phone 4. Because Product 1 is the least expensive core, it would be an excellent substitute for Product 4. Accordingly, as presented in Table 7, the optimal take-back plan for Family 1 involves less take-back of working Product 4 along with more take-back of non-working Product 1.

In addition to the cost reduction in product take-back and parts procurement, increased revenue for reconditioning is also examined. Since other phones take the place of Phone 4 in providing parts for refurbishment, the company can keep some of the available Phone 4 for other purposes without sacrificing refurbishment. Specifically, the company can recover Product 4 by the second most profitable way; *i.e.* reconditioning, which increases the overall recovery revenue.

6. Conclusions

End-of-life management is regarded as a problem of multiple cores with commonality. In order to improve the profitability of end-of-life management, a manufacturer should make commonality decisions in product family design by considering their influences on product take-back and end-of-life recovery. To help manufacturers make the best decisions, an optimization method was developed for assessing product family designs for their profitability in end-of-life management. Using mixed integer programming, the model identifies an optimal strategy for product take-back and end-of-life recovery, thereby assessing the maximum profits for the product family during the end-of-life stage. The profit value can be used as a quantitative measure to evaluate product family design. By applying this method in the design stage, manufacturers can assess various product family designs and choose the best one.

An example with a smartphone family illustrates how to apply the proposed model and how it supports decision making in product family design. The study results demonstrate that product family design can be a means of improving the profitability of end-of-life management. When multiple products are designed to have common components, their profit outweighs the reference case products in which no components are shared. Moreover, the superiority is examined not only in the magnitude of profit, but also in the return on investment. The results also imply that the profit monotonically increases with the level of component sharing, but the increasing amount differs from case to case, depending on the shared parts. Finally, the results show that product family design has the potential to support a more environmentally conscious product recovery. The high-sharing smartphone family produces greater value for the company from the same amount of material. Also, it requires a smaller amount of new material to achieve the maximum profit.

The economies of scale in recovery operations can be incorporated in the model in the future. As component commonality increases, the necessary tools, the required worker skills, and the time required for set-up can decrease in various recovery operations (*i.e.* disassembly, conditioning, part purchasing, warehousing, and reassembly). However, in this article, the economies of scale are excluded from consideration by assuming unlimited facility capacity and by assuming constant unit cost for every operation. Uncertainty is also an important aspect because many parameters, which are assumed to be deterministic in this article, are stochastic in reality. This article is one of the first attempts to examine product family design from the end-of-life point of view, thus using mixed integer programming was a natural choice; it is simple and provides a great foundation for a variety of studies in the future. However, uncertainty in real-world decisions must be considered in the assessment to find an optimal family design that is robust in handling possible changes. Future work should include the development of a stochastic model that can deal effectively with such uncertainties. Finally, an integrated approach should be developed in the future that considers both end-of-life stage and design and manufacturing stages. Combining the proposed model with traditional family design approaches will lead to a more advanced framework.

Acknowledgements

This material is based upon the work supported by the National Science Foundation under Award No. 0726934. Any opinions, findings and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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Appendix

Table A1. Recovery cost and revenue.

Item	Weight (lb)	c^e	c^c	c^d	c^r	c^y	c^s	c^l	r^m	r^u	r^c	r^z	D^u	D^c	D^z
Product 1	0.2908	1.5	1.00	1.5	2.0	·	1	0.0058	0.7270	135.0	180.00	225	10,000	10,000	10,000
Product 2	0.2952	1.5	1.00	1.5	2.0	·	1	0.0059	0.7380	165.0	220.00	275	10,000	10,000	20,000
Product 3	0.2952	1.5	1.00	1.5	2.0	·	1	0.0059	0.7380	165.0	220.00	275	10,000	10,000	20,000
Product 4	0.2996	1.5	1.00	1.5	2.0	·	1	0.0060	0.7490	195.0	260.00	325	10,000	10,000	50,000
Front screen assembly	0.0960	·	0.50	0.5	1.5	56	·	0.0019	0.2400	70.0	91.00	112	20,000	20,000	40,000
Lower dock assembly	0.0268	·	0.50	0.5	1.5	12	·	0.0005	0.0670	15.0	19.5	24	20,000	20,000	20,000
Real panel assembly 1	0.0672	·	0.75	0.5	1.5	38	·	0.0013	0.1680	47.5	61.75	76	5000	5000	20,000
Real panel assembly 2	0.0672	·	0.75	0.5	1.5	38	·	0.0013	0.1680	47.5	61.75	76	5000	5000	15,000
Real panel assembly 3	0.0672	·	0.75	0.5	1.5	38	·	0.0013	0.1680	47.5	61.75	76	5000	5000	15,000
Real panel assembly 4	0.0672	·	0.75	0.5	1.5	38	·	0.0013	0.1680	47.5	61.75	76	5000	5000	10,000
Logic board 1	0.0408	·	1.00	·	·	80	·	0.0008	0.2040	100.0	130.00	·	5000	5000	·
Logic board 2	0.0452	·	1.00	·	·	100	·	0.0009	0.2260	125.0	162.50	·	10,000	10,000	·
Logic board 3	0.0496	·	1.00	·	·	140	·	0.0010	0.2480	175.0	227.50	·	5000	5000	·
Camera	0.0100	·	0.50	·	·	4	·	0.0002	0.0250	5.0	6.50	·	20,000	20,000	·
Battery	0.0500	·	0.50	·	·	6	·	0.0010	0.0750	7.5	9.75	·	20,000	20,000	·
Digitizer	0.0384	·	0.75	·	·	14	·	0.0008	0.0960	17.5	22.75	·	20,000	20,000	·
LCD screen	0.0384	·	0.50	·	·	22	·	0.0008	0.0960	27.5	35.75	·	20,000	20,000	·
Ear speaker	0.0096	·	0.50	·	·	4	·	0.0002	0.0240	5.0	6.50	·	20,000	20,000	·
Mid chassis frame	0.0096	·	0.50	·	·	12	·	0.0002	0.0240	15.0	19.50	·	20,000	20,000	·
Antenna	0.0038	·	0.50	·	·	4	·	0.0001	0.0095	5.0	6.50	·	20,000	20,000	·
Dock connector	0.0096	·	0.50	·	·	6	·	0.0002	0.0240	7.5	9.75	·	20,000	20,000	·
Loud speaker	0.0096	·	0.50	·	·	4	·	0.0002	0.0240	5.0	6.50	·	20,000	20,000	·
Microphone	0.0038	·	0.50	·	·	3.6	·	0.0001	0.0095	4.5	5.85	·	20,000	20,000	·
Rear casing 1	0.0384	·	0.50	·	·	30	·	0.0008	0.0960	37.5	48.75	·	5000	5000	·
Rear casing 2	0.0384	·	0.50	·	·	30	·	0.0008	0.0960	37.5	48.75	·	5000	5000	·
Rear casing 3	0.0384	·	0.50	·	·	30	·	0.0008	0.0960	37.5	48.75	·	5000	5000	·
Rear casing 4	0.0384	·	0.50	·	·	30	·	0.0008	0.0960	37.5	48.75	·	5000	5000	·
Headphone jack assembly 1	0.0096	·	0.50	·	·	5.6	·	0.0002	0.0240	7.0	9.10	·	15,000	15,000	·
Headphone jack assembly 2	0.0096	·	0.50	·	·	5.6	·	0.0002	0.0240	7.0	9.10	·	5000	5000	·
GPS antenna	0.0096	·	0.50	·	·	4	·	0.0002	0.0240	5.0	6.50	·	20,000	20,000	·
Vibrator	0.0096	·	0.50	·	·	4	·	0.0002	0.0240	5.0	6.50	·	20,000	20,000	·

Note: When applying these parameters to a reference case, demand parameters (*i.e.* D^u, D^c, and D^z) for shared parts are needed to change. For a product variant, the demand for each part is changed into (D/n), where n is the number of variants sharing the part in the family design. For example, the demand for used camera is 5,000 (= 20,000/4) for each phone in the reference case.