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Analytical Target Cascading in Automotive Vehicle Design

Target cascading in product development is a systematic effort to propagate the desired top-level system design targets to appropriate specifications for subsystems and components in a consistent and efficient manner. If analysis models are available to represent the consequences of the relevant design decisions, analytical target cascading can be formalized as a hierarchical multilevel optimization problem. The article demonstrates this complex modeling and solution process in the chassis design of a sport-utility vehicle. Ride quality and handling targets are cascaded down to systems and subsystems utilizing suspension, tire, and spring analysis models. Potential incompatibilities among targets and constraints throughout the entire system can be uncovered and the trade-offs involved in achieving system targets under different design scenarios can be quantified.
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Introduction

The product development process for complex artifacts is most effective when the required design tasks can be accomplished in a concurrent and consistent manner. Concurrency means that individual design tasks are conducted separately, and consistency means that key links identified among different design tasks are observed and enforced until the concurrent design process yields a final product. The target cascading process attempts to achieve this consistency and concurrency early in the development process [1,2]. The important specifications or ''targets'' for the entire system (as well as for each subsystem and component) are identified first, specifically those that will influence other parts of the system. These targets are then propagated or ''cascaded'' to the rest of the system and appropriate values are assigned for the expected performance of each element of the system. The actual design tasks are then executed locally for each individual element, and interaction with the rest of the system is revisited only when a target cannot be met. When the design decisions can be modelled analytically, the process can be formalized as a multi-level optimization problem referred to as analytical target cascading (ATC). The formulation and solution of this problem is a complex task. Much of the motivation for the work described in this article comes from a need to demonstrate how target cascading will work for a problem of realistic complexity, such as an automotive vehicle.

Multilevel optimization methods have been well studied [e.g., [3,4]]. Collaborative optimization [5,6] is particularly interesting in the present context. In this formulation design objectives in the subproblems attempt to minimize the discrepancy between the interaction (or interdisciplinary) variables and the targets, and should become zero at the optimum. Constraints in the original optimization problem are distributed in the subsystem optimization problems, and subproblem objectives become equality constraints at the system level. During iterations, subproblems may return different values for an interdisciplinary variable, which can cause convergence difficulties in that equality constraints at the system level are not satisfied [7]. Convergence difficulties are not uncommon for the coordination strategies needed to solve multilevel optimization problems. Though different from collaborative optimization, target cascading shares the idea of minimizing deviations between design problems to achieve consistency but can be shown to satisfy constraint qualifications [2]. In collaborative optimization, analysis models are decomposed at the same level and a coordination problem is defined on top of the bilevel modeling hierarchy. Without a convergent coordination strategy, it is not clear how to extend collaborative optimization to a multilevel hierarchy. In target cascading, a multilevel optimization problem is formulated to enable multidisciplinary decision making at multiple levels. The nonascent property of hierarchical overlapping coordination is utilized to demonstrate non-ascent of the ATC coordination [2,8,9]. In the present study, models are checked for feasibility and boundedness [10] and for constraint qualifications of the additional deviation constraints [11].

The next section reviews briefly the basic concepts in the analytical target cascading process. A chassis design problem is then outlined, its constituent models are developed, and the mathematical problem is posed. Solution of this problem shows how top-level targets can be cascaded to derive subsystem and component specifications. Such a capability is shown to be an effective early product development tool: trade-offs among desired top-level target values can be quantitatively assessed, while incompatibilities can be uncovered and traced to design specifications or bounds at the subsystem and component levels.

Some Basic Concepts in Target Cascading

The reader is referred to Kim et al. [1] and Kim [2] for a complete explanation of generic ATC formulations. Here we draw attention to the distinction between the design and analysis models with which the hierarchy is constructed, and give the mathematical form of the ATC problem.

Modeling Hierarchy. The reader may refer to the IEEE Standards for multilevel systems engineering concepts for further description of partitioned design elements [12]. A complex problem, such as vehicle design, can be partitioned into a multilevel hierarchical structure. Two types of models exist in the modeling hierarchy of the ATC process: *optimal design models P* and *analysis models r* [1]. Optimal design models call analysis models to evaluate vehicle, system, subsystem and component responses. Thus, analysis models take design variables and parameters, as well as lower level responses, and return responses for design problems. A response is defined as an output from an analysis model, and a linking variable is defined as a design variable common between two or more design problems.

To represent the hierarchy of the partitioned design problem, the set of elements E_i is defined at each level i, in which all the elements of the level are included. For each element j in the set E_i, the set of children C_{ij} is defined, which includes the elements of the set E_{i+1} that are children of the element. An illustrative

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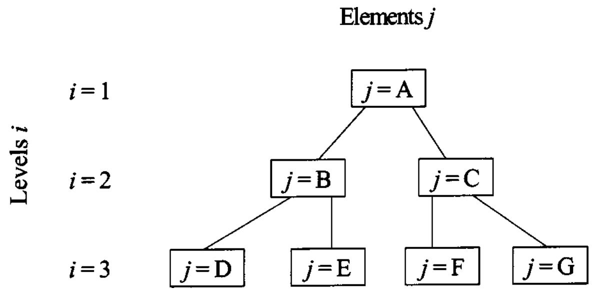

Fig. 1 Example of hierarchically partitioned optimal design problem

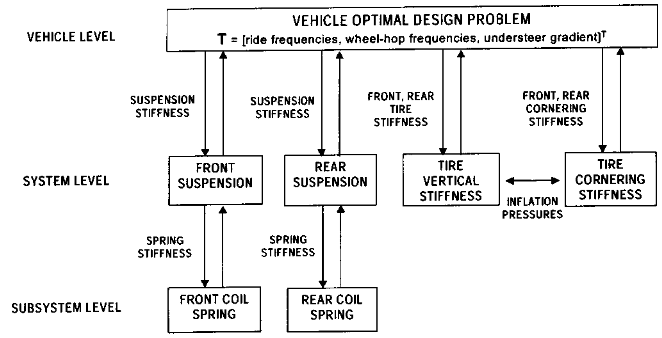

Fig. 3 SUV chassis design problem structure

example is presented on Fig. 1: At level $i=2$ of the partitioned problem we have $E_2=\{B,C\}$, and for element "B" at that level we have $C_{2B}=\{D,E\}$.

Figure 2 shows interactions between analysis models and design models at the system level. Targets for system responses and system linking variables $\mathbf{R}_s^U$ and $\mathbf{y}_s^U$ are passed down from the vehicle level. After solving the system design problem, target values for system responses and system linking variables $\mathbf{R}_s^L$ and $\mathbf{y}_s^L$ are passed up to the vehicle level. Likewise, for subsystem 1, $\mathbf{R}_{ss1}^U$ and $\mathbf{y}_{ss}^U$ are passed down as targets from the system-level design problem, whereas $\mathbf{R}_{ss1}^L$ and $\mathbf{y}_{ss1}^L$ are returned to the system level. Here $\mathbf{y}_{ss}^U$ is the same for all subsystem problems as it is calculated at the system level and cascaded as a target to subsystem problems. Responses from subsystem 1, $\mathbf{R}_{ss1}$, system local design variables $\tilde{\mathbf{x}}_{s1}$, and system linking variables $\mathbf{y}_{s1}$ are input to the analysis model r_{s1}, whereas system responses $\mathbf{R}_{s1}$ are returned as output.

Mathematical Problem Statement of the Design Problem. The original design problem, in a vehicle context, can be stated as follows: find a design that minimizes the deviations between the overall design targets and responses, while satisfying all constraints. Alternatively, determine the values of vehicle, system, subsystem and component parameters that minimize the deviation of vehicle responses from vehicle targets. The original design problem P_0 is formally stated in Eq. (1).

The objective is defined as the discrepancy between the target $\mathbf{T}$ and the response $\mathbf{R}$ obtained from the analysis model $r(\mathbf{x})$; $\mathbf{g}$ and $\mathbf{h}$ are inequality and equality design constraint vectors with sizes m_i, m_e, and the design variable $\mathbf{x}$ is defined within lower and upper bounds, $\mathbf{x}^{\min}$ and $\mathbf{x}^{\max}$.

$$P_0:\text{Minimize}_{\mathbf{x}} \ \|\mathbf{T}-\mathbf{R}\|$$

$$\text{where} \ \mathbf{R}=r(\mathbf{x})$$

$$\text{subject to}$$

$$g_i(\mathbf{x})\leq 0 \quad i=1,\ldots,m_i$$

$$h_j(\mathbf{x})=0 \quad j=1,\ldots,m_e$$

$$x_k^{\min}\leq x_k\leq x_k^{\max} \quad k=1,\ldots,n \tag{1}$$

A Target Cascading Process for Vehicle Ride and Handling

In this section we give an overview of an ATC model for the chassis system of a typical sport-utility vehicle (SUV) aimed at establishing vehicle ride and handling targets. The model is obviously simplified but retains sufficient complexity to be realistic. Figure 3 gives a schematic of the information flow in the vehicle design problem structure. Each block indicates an optimal design model where design decisions are made to achieve minimum deviation from the targets. Each design model calls one or more analysis models to evaluate the current design. The vehicle-level design problem contains two analysis models, a "half-car" model and a "bicycle" model. System-level analysis models for the front and rear suspensions are multibody-dynamics models of short-long arm (SLA) suspensions [13]. The tire models call the tire stiffness equations described in Wong [14].

The following vehicle-level targets for handling and ride quality are prescribed:

- first natural frequency of front and rear suspension (ω_{sf}, ω_{sr})
- second natural frequency (wheel hop frequency) of front and rear suspension (ω_{tf}, ω_{tr})
- understeer gradient (k_{us})

These five quantities constitute the target vector, for which the half-car and bicycle analysis models generate responses. The computed variable values are then cascaded to the system-level design problem as targets. For example, the front suspension stiffness is changed to achieve the desired first natural frequency for the front

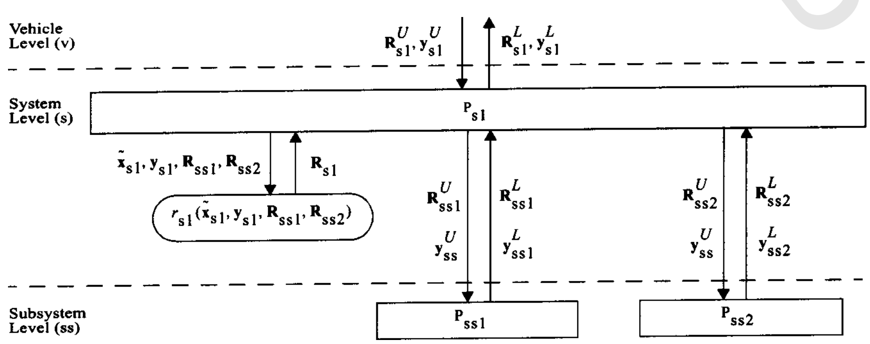

Fig. 2 Flows from/into the system-level design problem

suspension. Once an optical value of the stiffness is found at the vehicle design problem, that value becomes a *target* value at the system-level design problem, in which the suspension design variables (coil spring stiffness and free length) are altered to achieve a suspension configuration with a stiffness as close to the cascaded target value as possible. The computed values of the variables, such as the coil spring stiffness that gives the optimal suspension stiffness, are then cascaded to the subsystem level as targets. The spring subsystem variables are optimized to achieve minimal deviation from the targets assigned for the coil spring stiffness.

Similarly, optimal tire stiffness and cornering stiffnesses calculated at the vehicle level become targets at the system level, where system-level variables (tire inflation pressure) are changed to meet the stiffness targets. In tire design models for vertical and cornering stiffnesses, the inflation pressure is common, i.e., the inflation pressure is a linking variable.

Once the vehicle design targets are cascaded down to the lowest level, the resulting design information must then be passed back to higher levels, up to the top level. In general, it will not be possible to achieve the target values exactly in each design problem, due to constraints and variable bounds or due to lower level responses. For example, the front suspension stiffness obtained from the system-level optimization problem might not match the target value from the vehicle level due to constraints on coil spring free length and stiffness. Similarly, upon cascading the desired coil spring stiffness to the coil spring component design problem, packaging or fatigue constraints might result in spring stiffnesses deviating from the specified target value. Deviation in spring coil stiffness will subsequently result in a deviation of the overall suspension stiffness, which in turn will affect the first ride frequency of the vehicle. Thus an iterative process working in both a top-down and a bottom-up fashion will lead to a consistent design or uncover potential incompatibilities among overall system responses, targets, and element parameters.

Mathematical Problem Statement and Model Development

The full ATC model is presented in this section. At each level, we present the general form of the ATC model and then its instantiation to the problem at hand. The ATC process applied in the early stages of product development does not require high fidelity models. Rather, it requires models that capture the influence of those design variables and responses in each system element which would affect other parts of the system. Indeed finding models of *appropriate* fidelity is a practical challenge in the execution of the ATC process.

Vehicle Level. At the top level of the vehicle hierarchy the problem is stated as follows:

$$P_\nu: \ \text{Minimize}_{\tilde{\mathbf{x}}_\nu, \mathbf{y}_s, \mathbf{R}_s, \varepsilon_{\mathbf{R}}, \varepsilon_{\mathbf{y}}} \|\mathbf{R}_\nu - \mathbf{T}_\nu\| + \varepsilon_{\mathbf{R}} + \varepsilon_{\mathbf{y}}$$
$$\text{where} \quad \mathbf{R}_\nu = r_\nu(\mathbf{R}_s, \tilde{\mathbf{x}}_\nu)$$
$$\text{subject to}$$
$$\sum_{k \in C_\nu} \|\mathbf{R}_{s,k} - \mathbf{R}_{s,k}^L\| \le \varepsilon_{\mathbf{R}}$$
$$\sum_{k \in C_\nu} \|\mathbf{y}_s - \mathbf{y}_{s,k}^L\| \le \varepsilon_{\mathbf{y}}$$
$$\mathbf{g}_\nu(\mathbf{R}_\nu, \tilde{\mathbf{x}}_\nu) \le \mathbf{0}, \quad \mathbf{h}_\nu(\mathbf{R}_\nu, \tilde{\mathbf{x}}_\nu) = \mathbf{0}$$
$$\tilde{\mathbf{x}}_\nu^{\min} \le \tilde{\mathbf{x}}_\nu \le \tilde{\mathbf{x}}_\nu^{\max} \tag{2}$$

where $C_\nu = \{k_1, \ldots, k_{c_\nu}\}$, c_ν is the number of child elements of the vehicle-level problem and $\mathbf{R}_s = (\mathbf{R}_{s,1}, \ldots, \mathbf{R}_{s,c_\nu})$, $\mathbf{R}_s = \mathbf{R}_{s,1} \cup \ldots \cup \mathbf{R}_{s,c_\nu}$ and $\mathbf{R}_{s,i} \cap \mathbf{R}_{s,j} = \emptyset$ for $i \neq j$. The objective that minimizes deviation between design targets $\mathbf{T}_\nu$ and vehicle responses $\mathbf{R}_\nu$ is modified by adding deviation tolerances $\varepsilon_{\mathbf{R}}$ and $\varepsilon_{\mathbf{y}}$ to coordinate values of the responses from the system, $\mathbf{R}_s$, and the system linking variables, $\mathbf{y}_s$. At convergence, the deviation tolerance becomes zero as the system linking variables converge to the same values for the different systems. The values of the system responses match $\mathbf{R}_s^L$, where $\mathbf{R}_s^L$ is the target response calculated at the system optimal design problem. Finally, $\mathbf{g}_\nu$ and $\mathbf{h}_\nu$ are inequality and equality design constraints at the vehicle level, subsets of the original constraints $\mathbf{g}$ and $\mathbf{h}$.

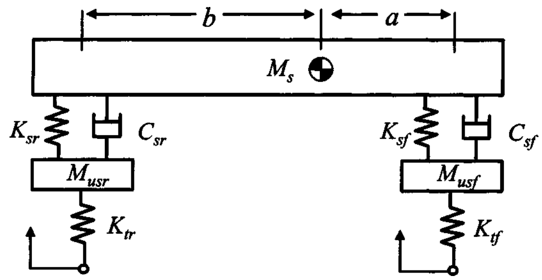

Fig. 4 Half-car model

The four ride quality targets involve the half-car model of Fig. 4. The target frequencies can be calculated in closed form as functions of sprung mass (M_s), front and rear unsprung masses (M_{usf}, M_{usr}), and suspension stiffnesses. The sprung and unsprung masses are assumed to be prescribed *a priori*, and are fixed design parameters. The vehicle body is treated as a single rigid body mass. Table 1 gives a summary of the vehicle-level variables, responses, and system-level linking variables and responses corresponding to the ATC formulation at the vehicle level in Eq. (2). The first natural frequencies of the suspensions are primarily affected by changing the front and rear suspension stiffnesses K_{sf}, K_{sr}, and to a lesser extent by modifying the distances a and b from the center of gravity to the axles. In the half-car model, front and rear damping coefficients C_{sf}, C_{sr} are parameters.

The handling target is the understeer gradient k_{us}, a measure of the magnitude and direction of the steering input for a vehicle to track a curve of constant radius ρ with forward velocity u and forward steer angle δ_f. For the purpose of understeer analysis, it is convenient to represent the vehicle by the bicycle model shown in Fig. 5. The understeer gradient is a function of a and b and of the front and rear tire lateral cornering stiffnesses $C_{\alpha f}$ and $C_{\alpha r}$.

The ATC design problem at the vehicle level is stated as follows.

$$P_\nu: \quad \text{Minimize} \|\omega_{sf} - \omega_{sf}^U\| + \|\omega_{sr} - \omega_{sr}^U\| + \|\omega_{tf} - \omega_{tf}^U\| + \|\omega_{tr} - \omega_{tr}^U\| + \|k_{us} - k_{us}^U\| + \varepsilon_{\mathbf{R}} + \varepsilon_{\mathbf{y}}$$
$$\text{with respect to}$$
$$(\omega_{sf}, \omega_{sr}, \omega_{tf}, \omega_{tr}, k_{us}, a, b)$$
$$(K_{sf}, K_{sr}, K_{tf}, K_{tr}, C_{\alpha f}, C_{\alpha r}, P_{if}, P_{ir}, \varepsilon_{\mathbf{R}}, \varepsilon_{\mathbf{y}})$$
$$\text{where}$$
$$\omega_{sf} = \sqrt{\frac{K_{sf}}{M_{sf}}}, \quad \omega_{sr} = \sqrt{\frac{K_{sr}}{M_{sr}}}, \quad \omega_{tf} = \sqrt{\frac{K_{tf}}{M_{usf}}},$$

Table 1 Summary of responses and variables at the vehicle level

Design problem	P_ν
Responses ($\mathbf{R}_\nu$)	ω_{sf}, ω_{sr}, ω_{tf}, ω_{tr}, k_{us}
Local variables ($\tilde{\mathbf{x}}_\nu$)	a, b
System-level linking variables ($\mathbf{y}_s$)	P_{if}, P_{ir}
Responses from system level ($\mathbf{R}_s$)	K_{sf}, K_{sr}, K_{tf}, K_{tr}, $C_{\alpha f}$, $C_{\alpha r}$

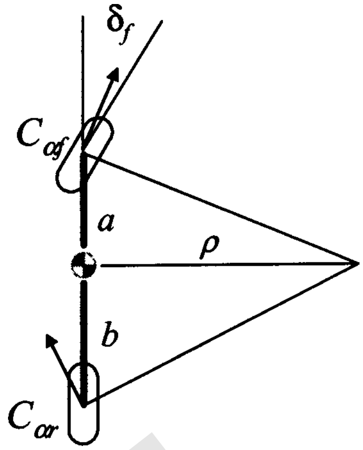

Fig. 5 Cornering of a bicycle model

$$\omega_{tr}=\sqrt{\frac{K_{tr}}{M_{usr}}},\qquad k_{us}=\frac{Mb}{LC_{\alpha f}}-\frac{Ma}{LC_{\alpha r}}, \tag{3}$$

subject to

$$\|K_{sf}-K_{sf}^{L}\|+\|K_{sr}-K_{sr}^{L}\|+\|K_{tf}-K_{tf}^{L}\|+\|K_{tr}-K_{tr}^{L}\|+\|C_{\alpha f}-C_{\alpha f}^{L}\| +\|C_{\alpha r}-C_{\alpha r}^{L}\|\leq\varepsilon_{\mathbf{R}},$$

$$\frac{1}{2}((P_{if}-P_{if}^{L}|_{vert})^2+(P_{if}-P_{if}^{L}|_{corn})^2)+\frac{1}{2}((P_{ir}-P_{ir}^{L}|_{vert})^2 +(P_{ir}-P_{ir}^{L}|_{corn})^2)\leq\varepsilon_{\mathbf{y}},$$

$$a^{\min}\leq a\leq a^{\max},\qquad b^{\min}\leq b\leq b^{\max}$$

Target Cascading at the System Level. At the system level the j^{th} problem is stated as in Eq. (4):

$$\mathbf{P}_{s,j}:\quad \text{Minimize}\quad \|\mathbf{R}_{s,j}-\mathbf{R}_{s,j}^{U}\|+\|\mathbf{y}_{s,j}-\mathbf{y}_{s,j}^{U}\|+\varepsilon_{\mathbf{R}}+\varepsilon_{\mathbf{y}}$$
$$\text{with respect to}\quad \tilde{\mathbf{x}}_{s,j},\mathbf{y}_{s,j},\mathbf{y}_{ss},\mathbf{R}_{ss},\varepsilon_{\mathbf{R}},\varepsilon_{\mathbf{y}}$$
$$\text{where}\quad \mathbf{R}_{s,j}=r_{s,j}(\mathbf{R}_{ss},\tilde{\mathbf{x}}_{s,j},\mathbf{y}_{s,j})$$
$$\text{subject to}$$
$$\sum_{k\in C_{s,j}}\|\mathbf{R}_{ss}-\mathbf{R}_{ss,k}^{L}\|\leq\varepsilon_{\mathbf{R}}$$
$$\sum_{k\in C_{s,j}}\|\mathbf{y}_{ss}-\mathbf{y}_{ss,k}^{L}\|\leq\varepsilon_{\mathbf{y}}$$
$$\mathbf{g}_{s,j}(\mathbf{R}_{s,j},\tilde{\mathbf{x}}_{s,j},\mathbf{y}_{s,j})\leq\mathbf{0},\qquad \mathbf{h}_{s,j}(\mathbf{R}_{s,j},\tilde{\mathbf{x}}_{s,j},\mathbf{y}_{s,j})=\mathbf{0}$$
$$\tilde{\mathbf{x}}_{s,j}^{\min}\leq\tilde{\mathbf{x}}_{s,j}\leq\tilde{\mathbf{x}}_{s,j}^{\max},\qquad \mathbf{y}_{s,j}^{\min}\leq\mathbf{y}_{s,j}\leq\mathbf{y}_{s,j}^{\max} \tag{4}$$

where $C_{s,j}=\{k_1,\ldots,k_{c_{s,j}}\}$, $c_{s,j}$ is the number of child element of system-level problem and $\mathbf{R}_{ss}=(\mathbf{R}_{ss,1},\ldots,\mathbf{R}_{ss,c_{s,j}})$. The objective function minimizes the discrepancy between current system level responses $\mathbf{R}_{s,j}$ and the targets set at the upper (vehicle) level $\mathbf{R}_{s,j}^{U}$, as well as between system linking variables $\mathbf{y}_{s,j}$ and the targets set at the vehicle level $\mathbf{y}_{s,j}^{U}$. Therefore, $\mathbf{R}_{s,j}^{U}$ and $\mathbf{y}_{s,j}^{U}$ are determined by solving Eq. (2). Target deviation tolerances are minimized to achieve consistent design with minimum discrepancies between the subsystem level responses $\mathbf{R}_{ss}$ and the target responses $\mathbf{R}_{ss}^{L}$ from the subsystem design problem, as well as between the subsystem level linking variables $\mathbf{y}_{ss}$ and the target values $\mathbf{y}_{ss}^{L}$ from the subsystem design problem. Since the system level is located in the middle of the overall hierarchy, this formulation is the most comprehensive, capturing all interactions, through linking variables, target responses from the lower level (superscript L), and target responses from the upper level (superscript U).

In the current study, there exist four design models at the system level: models for the front and rear suspensions, and tire models for vertical and cornering stiffness (Fig. 3). The ATC system-level design problem for the front suspension model is stated as follows.

$$\mathrm{P}_{s1}:\quad \text{Minimize}\quad \|K_{sf}-K_{sf}^{U}\|+\varepsilon_{\mathbf{R}}$$
$$\text{with respect to}\quad (Z_{sf},K_{Lf},K_{Bf},L_{0f},\varepsilon_{\mathbf{R}})$$
$$\text{where}\quad K_{sf}=\text{AutoSim}(Z_{sf},K_{Lf},K_{Bf},L_{0f})$$
$$\text{subject to}$$
$$\|K_{Lf}-K_{Lf}^{L}\|+\|K_{Bf}-K_{Bf}^{L}\|+\|L_{0f}-L_{0f}^{L}\|\leq\varepsilon_{\mathbf{R}}$$
$$K_{Lf}^{\min}\leq K_{Lf}\leq K_{Lf}^{\max}$$
$$K_{Bf}^{\min}\leq K_{Bf}\leq K_{Bf}^{\max}$$
$$L_{0f}^{\min}\leq L_{0f}\leq L_{0f}^{\min}$$
$$Z_{sf}^{\min}\leq Z_{sf}\leq Z_{sf}^{\max} \tag{5}$$

For a given target value for suspension stiffness from the vehicle ATC problem in Eq. (3), the objective is to minimize the discrepancy between target and response. As there is no linking variable at the subsystem level, the tolerance term for minimizing the linking variable deviation is not included in the objective function. Besides the original variable bound constraints for suspension design, additional deviation constraints from the subsystem level are included in the constraint set. Deviations for subsystem level responses $K_{Lf}^{L},K_{Bf}^{L},L_{0f}^{L}$ are constrained within tolerance. The analysis model is a stand-alone, executable PC file generated using AutoSim [13,15]. The model is completely parametric and dependent on user inputs for key suspension properties and geometry. In other words, by changing some values in the input file, the suspension model can be used to evaluate different design specification. The ATC design problem for rear suspension model P_{s2} is the same as the one for the front except that it has different variable bounds.

The tire was represented as a single spring in the half-car model in the vehicle. At the system level, two different aspects of the

Table 2 Summary of responses and variables at the system level

Design problem	P_{s1} Front Suspension Problem	P_{s2} Rear Suspension Problem	P_{s3} Tire Vertical Stiffness Problem	P_{s4} Tire Cornering Stiffness Problem
Responses ($\mathbf{R}_s$)	K_{sf}	K_{sr}	K_{tf}, K_{tr}	$C_{\alpha f}$, $C_{\alpha r}$
Local variables ($\tilde{\mathbf{x}}_s$)	Z_{sf}	Z_{sr}	N/A	N/A
System-level linking variables ($\mathbf{y}_s$)	N/A	N/A	P_{if}, P_{ir}	P_{if}, P_{ir}
Responses from subsystem level ($\mathbf{R}_{ss}$)	K_{Lf}, K_{Bf}, L_{0f}	K_{Lr}, K_{Br}, L_{0r}	N/A	N/A

same tire analysis model, vertical and cornering, are considered, and for each aspect an ATC design problem is formulated.

The design models for the vertical and cornering tire stiffness are described in the following equations Eq. (6) and Eq. (7).

$$P_{s3}:\ \text{Minimize}\quad \|K_{tf}-K_{tf}^U\|+\|K_{tr}-K_{tr}^U\|+\|P_{if}-P_{if}^U\| + \|P_{ir}-P_{ir}^U\|$$

$$\text{with respect to}\quad (K_{tf},K_{tr},P_{if},P_{ir})$$

where

$$K_{tf}=0.9((0.1839P_{if}-9.2605)F_m+110119)$$

$$K_{tr}=0.9((0.1839P_{ir}-9.2605)F_m+110119)$$

$$F_m=\frac{9.81Mb}{a+b} \tag{6}$$

subject to

$$P_{if}^{\min}\leqslant P_{if}\leqslant P_{if}^{\max}$$

$$P_{ir}^{\min}\leqslant P_{ir}\leqslant P_{ir}^{\max}$$

$$P_{s4}:\ \text{Minimize}\quad \|C_{\alpha f}-C_{\alpha f}^U\|+\|C_{\alpha r}-C_{\alpha r}^U\|+\|P_{if}-P_{if}^U\| + \|P_{ir}-P_{ir}^U\|$$

$$\text{with respect to}\quad (C_{\alpha f},C_{\alpha r},P_{if},P_{ir})$$

where

$$C_{\alpha f}=F_m(-2.668\times10^{-6}P_{if}^2+1.605\times10^{-3}P_{if} -3.86\times10^{-2})\,\frac{180}{\pi}$$

$$C_{\alpha r}=F_m(-2.668\times10^{-6}P_{ir}^2+1.605\times10^{-3}P_{ir}-3.86\times10^{-2})\,\frac{180}{\pi}$$

$$F_m=\frac{9.81Mb}{a+b}$$

subject to

$$P_{if}^{\min}\leqslant P_{if}\leqslant P_{if}^{\max}$$

$$P_{ir}^{\min}\leqslant P_{ir}\leqslant P_{ir}^{\max} \tag{7}$$

In the tire models, the objective function is to minimize deviations for the front and rear tire stiffnesses (vertical K_{tf},K_{tr} or cornering $C_{\alpha f},C_{\alpha r}$) and the tire inflation pressures for the front and rear P_{if},P_{ir} subject to variable bound constraints for the tire inflation pressures for the front and rear. The tire infltion pressures for the front and rear are linking variables that are both included in Eq. (6) and Eq. (7). The stiffness of the tire in the vertical direction is a function of the inflation pressures and the datum vertical load on the tire F_m that is a function of the tire distances a and b and the mass of the vehicle M. The inflation pressures for the front and rear tires are linking variables that are coordinated at the vehicle level in Eq. (3).

Target Cascading at the Subsystem Level. At the subsystem level the j^{th} problem is stated in Eq. (8): Minimize the deviations for subsystem responses and subsystem level linking variables subject to subsystem design constraints. Formally,

$$P_{ss,j}:\quad \text{Minimize}_{\tilde{\mathbf{x}}_{ss,j},\mathbf{y}_{ss,j}}\|\mathbf{R}_{ss,j}-\mathbf{R}_{ss,j}^U\|+\|\mathbf{y}_{ss,j}-\mathbf{y}_{ss,j}^U\|$$

$$\text{where}\quad \mathbf{R}_{ss,j}=r_{ss,j}(\tilde{\mathbf{x}}_{ss,j},\mathbf{y}_{ss,j})$$

subject to

$$\mathbf{g}_{ss,j}(\mathbf{R}_{ss,j},\tilde{\mathbf{x}}_{ss,j},\mathbf{y}_{ss,j})\leqslant\mathbf{0},\mathbf{h}_{ss,j}(\mathbf{R}_{ss,j},\tilde{\mathbf{x}}_{ss,j},\mathbf{y}_{ss,j})=\mathbf{0} \tag{8}$$

$$\tilde{\mathbf{x}}_{ss,j}^{\min}\leqslant\tilde{\mathbf{x}}_{ss,j}\leqslant\tilde{\mathbf{x}}_{ss,j}^{\max},\quad \mathbf{y}_{ss,j}^{\min}\leqslant\mathbf{y}_{ss,j}\leqslant\mathbf{y}_{ss,j}^{\max}$$

At the bottom of the model hierarchy, subsystem design variables are input to the analysis models $r_{ss,j}$ returning responses to the subsystem level as output. In Eq. (8) the objective is to minimize the deviations between the subsystem responses $\mathbf{R}_{ss,j}$ and the targets set at the system level $\mathbf{R}_{ss,j}^U$, as well as between the subsystem linking variables $\mathbf{y}_{ss,j}$ and the targets from the system level $\mathbf{y}_{ss,j}^U$. Target deviation tolerance constraints are not introduced in Eq. (8) because there are no lower level design models that need to be coordinated.

At the subsystem level below the suspension model, the front and rear coil spring design models minimize the difference between target coil spring stiffness and the response generated by the spring design analysis model. The coil spring design model attempts to minimize an objective function that is a weighted sum of the difference between target and actual linear spring stiffness, bending stiffness, and free length, while satisfying the following constraints [16].

- Maximum shear stress with safety factor must not be exceeded;
- spring must not fail in fatigue;
- coil diameter and wire diameter must fall within specified bounds;
- wire diameter must be greater than the pitch;
- wire diameter to coil diameter ratio must be within limits;
- spring must not be fully compressed at maximum suspension travel.

The detailed equations for coil spring design including the above constraints are given in the following Eq. (9). Target values for linear spring stiffness K_L, bending stiffness K_B, and free length L_0 are cascaded down from the system level. Subsystem design variables are the wire diameter d_f, coil diameter D_f, and pitch p_f. Once the optimal design is found, the updated target values are returned to the system level. The design model for the subsystem front coil spring design is given in the following Eq. (9),

$$P_{\text{sub1}}:\text{Minimize}\ \|K_{Lf}-K_{Lf}^U\|+\|K_{Bf}-K_{Bf}^U\|+\|L_{0f}-L_{0f}^U\|$$

$$\text{with respect to}(D_f,d_f,p_f)$$

where

$$K_{Lf}=\frac{Gd_f^4}{8D_f^3\left(\frac{L_{0f}-3d_f}{p_f}\right)},K_{Bf}=\frac{Egd_f^4}{16D_f(2G+E)}$$

subject to

$$(F_a+F_m)\times\left(\frac{8D_f}{\pi d^3}+\frac{4}{\pi d_f^2}\right)-\frac{S_{su}}{n_s}\leqslant0$$

$$n_f-\left(\frac{S_{su}S_{se}\pi d_f^3}{8D_f}\right)\bigg/\left(\left(\frac{4D_f}{d_f}+2\right)\bigg/\left(\frac{4D_f}{d_f}-3\right)F_aS_{su} +\left(\frac{2D_f}{d_f}+1\right)\bigg/\left(\frac{2D_f}{d_f}\right)F_mS_{se}\right)\leqslant0$$

$$p_f-d_f\leqslant0$$

$$D_f^{\min}\leqslant D_f\leqslant D_f^{\max}$$

$$d_f^{\min}\leqslant d_f\leqslant d_f^{\max} \tag{9}$$

where L_0 is the spring free length, G is the modulus of rigidity of spring material, n_S is the factor of safety in shear, S_{su} is the maximum allowable shear stress, S_{se} is the fatigue endurance limit, and F_a, F_m are the alternating and mean components of spring load, respectively.

Table 3 Vehicle targets and vehicle responses

Target	Desired value	Scenario A	Scenario B	Scenario C
Front suspension first natural frequency ω_{sf} [Hz]	1.20	1.18	1.17	1.17
Rear suspension first natural frequency ω_{sr} [Hz]	1.44	1.56	1.56	1.49
Front suspension wheel hop frequency ω_{tf} [Hz]	12.00	11.94	11.99	11.97
Rear suspension wheel hop frequency ω_{tr} [Hz]	12.00	12.11	12.07	12.08
Understeer gradient k_{us} [rad/m/s^2]	0.00719	0.0056	0.0065	0.0066

This concludes our discussion of the formal statement of the problem. The next section discusses the results of the process and explores the effects of changing target values, target weights, and design constraints.

Design Scenario Analysis

The computational process used to solve the ATC problem in this study was as follows (Fig. 3): First, the top level vehicle design problem was solved and system level targets were cascaded. Second, four system-level problems were solved independently based on the targets assigned from the top level. Third, subsystem-level problems for the front/rear coil spring design were solved. Based on the subsystem-level responses, system-level design problems for front/rear suspension design were solved again and all the system-level responses and linking variables from the four system design problems were fed back to the top level, completing one iteration. This process corresponds to one of the convergent solution sequences that are proven to be convergent to the optimal solution [17]. Iterations were terminated when the deviation terms became smaller than tolerance ε. Typically this was achieved within ten iterations. Local convergence characteristics should be further investigated in terms of the convergence speed and the value of tolerance ε.

In principle, the final results upon convergence of the target cascading algorithm depend on the relative weights assigned to the deviations from targets, on the target values themselves, and on the constraint bounds. In the ATC formulations in Eq. (2), Eq. (4), and Eq. (8), the deviations terms in the objectives are equally weighted given the assumption that they are scaled. In a multidisciplinary design exercise, decisions about the relative importance of each target are made *a priori* and may require adjustment depending on the degree and nature of their incompatibility. High level discussion subsequent to unsatisfactory target achievement may also result in constraint relaxation and thus a different design space. These issues are examined below in light of the chassis design problem results.

Design Scenario A: Equally Weighted Ride and Handling. Targets. The baseline study attempted to satisfy all design departments involved by assigning equal weight for each ride and handling target after scaling. Deviation quantities were scaled to the same order of magnitude to provide a meaningful comparison. Equal weights were used.

The target values and responses from the baseline study are given in Table 3. Figure 6 shows normalized comparisons of targets and responses, where "1" denotes an exact match, greater than 1 denotes exceeding the target, and less than 1 denotes not reaching the target. Exceeding the target does not necessarily mean better design, i.e., better than expected, because responses are normalized and the closer the response value is to 1, the better the target match. The optimal design from scenario A is given in Table 4 to Table 10. It is shown that ATC yielded a consistent design such that for a given design quantity (like the front suspension stiffness) that was cascaded down from level i to level $(i+1)$ as a target, the response for that design quantity from the analysis model at level $(i+1)$ matched the target closely, within a tolerance. Similarly, linking variables converged to a single value within tolerance for each system they affected. If the tolerances were tightened, then the responses and linking variables would have matched more closely. Note from the tables that some of the variables reach their optimal values at lower or upper bounds; for example, the lower bound for the rear linear coil spring stiffness is active. This suggests that if the variable bounds were relaxed, the overall response would be changed for better achievement of targets. The final response values matched the targets closely, with the exception of the understeer gradient. Understeer gradient is partly a function of the distances *a* and *b*, which were at their bounds.

In the following subsections, two different design scenarios B and C will be presented. Design scenario B uses different target weights, and design scenario C uses a modified design space. Indeed when the "design authority" encounters discrepancies in the achievement of certain design targets, there are two options to exercise: (1) increasing the weight for the targets with high discrepancies, or (2) changing the design space. The following design scenarios explore these two options.

Design Scenario B: Modification of Target Weights. Given the results of the baseline study in design scenario A, the design authority must assess the acceptability of the responses. If a certain response, for example the understeer gradient, is deemed too low, target cascading can be reapplied either with different target weights (i.e., different objective function) or with a different design space. In design scenario B, the target weight for understeer gradient was increased fivefold in an attempt to increase the contribution of the understeer gradient deviation term in the objective function. No changes were made to the feasible design space.

Responses after changing the target value are given in Table 3 and plotted in Fig. 6. In Fig. 6 the understeer gradient is compared to the same target value from the baseline study. In other words, ratio "1" for understeer gradient means a perfect match with the 0.00719 [rad/m/s^2] target value from design scenario A. Design scenario B, changing the target weight on the understeer gradient term, led to a better design in terms of understeer gradient target. Note that the rear ride frequency is overpredicted consistently. This frequency is dependent on the actively constrained rear suspension spring stiffness, which suggests that relaxing the fea-

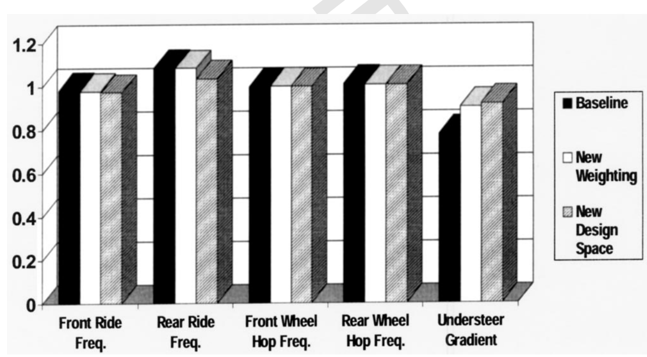

Fig. 6 Comparison between design scenario A (baseline), B and C: "1" represents exact target match

Table 4 Vehicle-level baseline design: Design scenario A

Vehicle design	Initial values	Optimal values	Lower bounds	Upper bounds
CG distance to front a [m]	1.32	1.25	1.25	1.39
CG distance to rear b [m]	2.38	2.45	2.31	2.45
Front suspension stiffness K_{sf} [N/mm]	40	41.3	13.13	60
Rear suspension stiffness K_{sr} [N/mm]	40	37.17	25.7	60
Front vertical tire stiffness K_{tf} [N/mm]	20	32.09	14.29	49.38
Rear vertical tire stiffness K_{tr} [N/mm]	20	33	12.56	34.55
Front cornering stiffness $C_{\alpha f}$ [N/rad/10^{-4}]	10	11.23	7.08	19.6
Rear cornering stiffness $C_{\alpha r}$ [N/rad/10^{-4}]	10	9.25	4.28	12.23

sible domain for the stiffness would lead to better achievement of the target. This is investigated in design scenario C.

Design Scenario C: Modification of Design Space. For design scenario C, target values were kept the same as in the baseline design scenario A. Instead, the variable bound for rear suspension coil spring stiffness was relaxed from 120 (N/mm) to 100 (N/mm). A hypothetical design authority, upon receiving feedback from the baseline design, would realize that the targets assigned for each department are not achievable within the initial design space. Also, in the case of boundary optima, changing target values would not help produce a better design in terms of achieving targets closely. The designers might then be allowed to change the feasible space, material, or configuration.

In the current study the feasible space was changed by relaxing variable bounds for certain design variables. As a result, the response (rear suspension first natural frequency) that had consistently exceeded the target value now has a response closer to the target value than in design scenario B. At the same time, the understeer gradient still has a response closer to the target value than in design scenario A. The lower bounds on some variables

Table 5 Front suspension system baseline design: Design scenario A

Front suspension system design	Initial values	Optimal values	Lower bounds	Upper bounds
Linear coil spring stiffness K_{Lf} [N/mm]	159	142.5	120	180
Spring free length L_{0f} [mm]	393.6	384.9	350	420
Spring bending stiffness K_{Bf} [N-mm/deg]	82500	79400	75000	85000
Overall suspension stiffness K_{sf} [N/mm]		41.32[a]	13.13	60
Suspension travel Z_{sf} [m]		0.0589	0.05	0.1

a. System response

Table 6 Rear suspension system baseline design: Design Scenario A

Rear suspension system design	Initial values	Optimal values	Lower bounds	Upper bounds
Linear coil spring stiffness K_{Lr} [N/mm]	159	120	120	180
Spring free length L_{0r} [mm]	393.6	417.2	350	420
Spring bending stiffness K_{Br} [N-mm/deg]	82500	85000	75000	85000
Overall suspension stiffness K_{sr} [N/mm]		37.2[a]	25.7	60
Suspension travel Z_{sr} [m]		0.0823	0.05	0.1

a. System response

Table 7 Front coil spring subsystem design: Design scenario A

Front coil spring subsystem design	Initial values	Optimal values	Lower bounds	Upper bounds
Wire diameter d_f [m]	0.0216	0.0232	0.005	0.03
Coil diameter D_f [m]	0.1507	0.18	0.05	0.2
Pitch p_f	0.0781	0.088	0.005	0.1
Linear coil spring stiffness K_{Lf} [N/mm]		142.6[a]	120	180
Spring bending stiffness K_{Bf} [N-mm/deg]		79407[a]	75000	85000

a. Subsystem response

Table 8 Rear coil spring subsystem design: Design scenario A

Rear coil spring subsystem design	Initial values	Optimal values	Lower bounds	Upper bounds
Wire diameter d_r [m]	0.0216	0.0237	0.005	0.03
Coil diameter D_r [m]	0.1507	0.18	0.05	0.2
Pitch p_r	0.0781	0.082	0.005	0.1
Linear coil spring stiffness K_{Lr} [N/mm]		120[a]	120	180
Spring bending stiffness K_{Br} [N-mm/deg]		85000[a]	75000	85000

a. Subsystem response

Table 9 Front coil spring subsystem design: Design scenario A

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Table 10 Rear coil spring subsystem design: Design scenario A

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are still active despite rear coil spring stiffness being relaxed, raising the possibility that simultaneous attainment of all targets is not feasible for this SUV chassis design exercise within the assumed ranges of design parameters.

Discussion. Both scenarios B and C led to a smaller discrepancy for the fifth target (understeer gradient) than the baseline design scenario A. Increasing target weight (scenario B) for the highest deviation response reduced the deviation significantly compared to that of the baseline design because more emphasis was put on the deviation term. Although some other responses then deviated more from the targets, changing design space (scenario C) improved the responses relevant to the active design constraints.

The design teams can learn from these scenarios that when there is a discrepancy for certain targets, it is critical to have alternative design options rather than assigning new target values. Alternative design options include changing target weights, changing the design space, designing with different materials or changing the design configuration.

The four system-level design problems could be solved in a parallel fashion, but in this study they were solved sequentially, maintaining independent solution processes for each problem. Comparing computational efficiency for each scenario was not considered in the current study.

Conclusion

Analytical target cascading provides a rich framework for addressing large-scale, multi-disciplinary system design problems with a multilevel structure. Responses, linking variables, and local variables capture interactions between design problems and analysis models. From a design viewpoint, the main benefit of the proposed approach for target cascading is reduction in large-scale product design cycle time, avoidance of design iterations late in the development process, and increased likelihood that physical prototypes will be closer to production quality. The main difficulty is obtaining the appropriate analysis models. The convergence issue in target cascading is further discussed in Kim [2] and Michelena et al. [17].

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Nomenclature

$C_{\alpha f}$ = front tire lateral cornering stiffness
$C_{\alpha r}$ = rear tire lateral cornering stiffness
C_{ij} = set of children elements; includes elements of the set E_{i+1} that are children of the element
E_i = set of elements in which all the elements at level i are included
K_B = coil spring bending stiffness
K_L = linear coil spring stiffness
K_{sf} = front suspension stiffness
K_{sr} = rear suspension stiffness
K_{tf} = front tire stiffness
K_{tr} = rear tire stiffness
L_0 = coil spring free length
P_{if} = front tire inflation pressure
P_{ir} = rear tire inflation pressure
P_o = original design optimization problem
P_v = vehicle-level target cascading (optimization) problem
P_s = system-level target cascading (optimization) problem
P_{ss} = subsystem-level target cascading (optimization) problem
$\mathbf{R}^L$ = target values for $\mathbf{R}$ propagated from a lower level
$\mathbf{R}^U$ = target values for $\mathbf{R}$ propagated from an upper level
$\mathbf{R}$ = responses computed by analysis models
$\mathbf{T}$ = design targets
a = distance from vehicle center of mass to front axle
b = distance from vehicle center of mass to rear axle
f = design objective
$\mathbf{g}$ = inequality constraints for the design problem
$\mathbf{h}$ = equality constraints for the design problem
k_{us} = understeer gradient
r = response function
u = vehicle forward velocity
$\mathbf{x}$ = vector of all design variables $(\tilde{\mathbf{x}},\mathbf{y})$
$\tilde{\mathbf{x}}$ = local design variables
$\mathbf{x}^{\min}$ = lower bound of $\mathbf{x}$
$\mathbf{x}^{\max}$ = upper bound of $\mathbf{x}$
$\mathbf{y}$ = linking design variables
$\mathbf{y}^L$ = target values for $\mathbf{y}$ propagated from a lower level
$\mathbf{y}^U$ = target values for $\mathbf{y}$ propagated from an upper level
Z_s = suspension deflection at jounce bumper contact
$\varepsilon_{\mathbf{R}}$ = target deviation tolerance for responses
$\varepsilon_{\mathbf{y}}$ = target deviation tolerance for linking variables
ρ = radius of track in bicycle model
ω_p = pitch natural frequency
ω_{sf} = first natural frequency of front suspension
ω_{sr} = first natural frequency of rear suspension
ω_{tf} = second natural frequency (wheel hop frequency) of front suspension
ω_{tr} = second natural frequency (wheel hop frequency) of rear suspension

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