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# Greedy robust wind farm layout optimization with feasibility guarantee

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**ABSTRACT**

This article proposes a method with light data requirements for generating robust wind farm layouts. Robustness in this work is quantified as the lowest energy conversion efficiency of the wind farm across all wind directions. A quadratic integer programming formulation for generating robustness-maximizing layouts is presented. Small instances of the proposed formulation can be solved to optimality using branch and bound. A modified greedy algorithm that guarantees solution feasibility with regards to inter-turbine safety distance is proposed to find solutions to larger problem instances. A series of experiments were conducted using real world wind data collected at two sites to demonstrate the trade-offs in power generation between robust layouts and power output maximizing layouts. The results show a loss of around 1.1% in hourly power generation in return for an increase in minimum power output of 1% to 45% across all directions for robust layouts generated in the experiments. The increase in robustness largely depends on the shape and orientation of the wind farm relative to the dominant wind direction, as well as the difference between the average wind speed at the site of the wind farm and rated wind speed of the turbines.

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Wind farm; layout optimization; robust; greedy algorithm

## 1. Introduction

Utility-scale wind power is generated at large wind farms consisting of horizontal axis wind turbines built on inland or offshore locations with high average wind speeds. The problem of determining positions of turbines within the wind farm is known as the layout optimization problem. Power output, or some power output related measure such as expected cost per unit energy, is a commonly used objective function in the layout optimization problem. The relationship between wind farm power output and turbine layout lies in the wake cones generated by moving turbine blades. As wind passes through a turbine's blades, a cone-shaped region of turbulent, slower moving air is created downstream of the turbine. Therefore, arranging the turbines in a manner that minimizes the extent of wake cone overlaps along wind directions with higher wind speeds would naturally lead to higher expected power output from the wind farm.

One of the earliest works on wind farm layout optimization was by Mosetti, Poloni, and Diviacco (1994), which used a genetic algorithm (Goldberg 1989) to find the power output maximizing layout of turbines over a finite set of possible turbine locations. The layout optimization problem then received renewed interest in the last decade as wind power generation started to increase in electricity grids around the world. Castro Mora *et al.* (2007), Zhang *et al.* (2012), Feng and Shen (2017), Bansal, Farswan, and Nagar (2018) and Wang *et al.* (2018) expanded the scope of the problem to include

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the number and configuration of turbines as decision variables in addition to turbine layout. Works by Serrano González *et al.* (2011), Saavedra-Moreno *et al.* (2011), Pillai *et al.* (2015) and Klein and Haugland (2017) added turbine layout related electrical and civil infrastructure costs to the problem formulation. Wind power generation and costs can be combined in a single objective function as cost per unit energy, or they can be treated as separate weighted objectives as demonstrated by Manjarres *et al.* (2014) and Rodrigues *et al.* (2016). There were also other works which added considerations such as land leasing costs (Chen and MacDonald 2014), complex terrain (Dobrić and Đurišić 2014; Song *et al.* 2015), and turbine fatigue (Réthoré *et al.* 2014) to the layout optimization problem. The reader can refer to works by Herbert-Acero *et al.* (2014) and Serrano González *et al.* (2014) for recent reviews of previous approaches to the layout optimization problem. The scope and complexity of the layout optimization problem has increased since the earliest works, but power output maximization has remained an important objective.

The actual performance of power output maximizing layouts depends on whether the probability models used in the layout optimization problem are accurate predictors of wind speeds and directions during the long operational lifetime of the wind farm. Inaccuracies in predictive wind modelling can be largely avoided by collecting as much on-site wind data as possible. Messac, Chowdhury, and Zhang (2012) showed that if the data collection period is much shorter than the wind farm's lifetime, yearly variations in wind speeds or directions could result in misleading predictive models and layouts that fail to capture as much wind energy as expected.

Turbine layouts that are less sensitive to wind prediction errors have been the subject of previous works in literature. Serrano González, Burgos Payán, and Riquelme Santos (2012) suggested an approach for reducing modelling error by considering multiple scenarios with different wind profiles when evaluating the expected profitability of the wind farm. However, the article did not describe how to generate these scenarios and their respective probabilities. Messac, Chowdhury, and Zhang (2012) proposed modelling not only the annual variation in wind speeds and directions, but also year-to-year variations as well. Having a predictive model of how wind directions and speeds will change in the long run can lead to better performing layouts, but building these predictive models requires significant amounts of wind data collected over multiple years. Song *et al.* (2016) developed a two stage algorithm that first finds a power output maximizing layout before performing local adjustments of turbine positions to minimize the number of turbines lined up along every wind direction. The two stage algorithm was able to increase significantly the minimum power output across all wind directions for the examples considered in the article, resulting in layouts that are less vulnerable to wind prediction errors, but the quality of the second stage solution could be constrained by the initial power output maximizing layout.

The concept of robust turbine layouts that are resilient to wind prediction errors is characterized in this work to be turbine layouts that are able to maintain a high level of efficiency, which is the wind farm's ability to convert incoming wind energy to electrical energy, regardless of whichever direction the wind is blowing. To be precise, let $t$ be the number of turbines in the wind farm. Suppose that the continuous range of wind directions is discretized into a finite set of directions indexed by the set $K$, and let $v_0$ be the average wind speed at the site of the wind farm. Then the robustness, $R_{t,K,v_0}$, of a turbine layout represented by the solution vector $\mathbf{x}$ is quantified in Equation (1). $F_{t,k,v_0}(\mathbf{x})$ in Equation (1), referred to as the directional power output, is the power output of the turbine layout $\mathbf{x}$ along wind direction $k \in K$, which has an ambient wind speed of $v_0$. The proposed definition has relatively light data requirements compared to previous works, requiring only an estimate of the average wind speed at the site of the wind farm, and makes no assumptions regarding future wind speed or direction probability distributions.

$$R_{t,K,v_0}(\mathbf{x}) := \arg\min_{k \in K} F_{t,k,v_0}(\mathbf{x}). \tag{1}$$

This article presents an integer programming formulation for generating turbine layouts that maximize robustness as defined in Equation (1). Turbine locations in the proposed formulation are limited to a finite set of possible points that can be located arbitrarily close to each other. It is also assumed that the number and type of turbines are fixed prior to the optimization step. A custom greedy algorithm that guarantees solution feasibility with regards to the minimum safety distance between turbines is presented for solving the proposed formulation.

The proposed problem formulation is introduced in Section 2. Section 3 describes the solution method in detail, and Section 4 contains the results of experiments aimed at highlighting the trade-offs between power output maximization and robustness in layout optimization. Having a robust layout comes at the price of less energy captured by the wind farm. The experiments use real world wind data collected over multiple years to highlight the extent of the trade-off between wind farm robustness and power output. Finally, conclusions and potential future work are discussed in Section 5.

## 2. Problem formulation

Problem formulations in layout optimization can be split into two categories, depending on the nature of the feasible space for turbine placement. The first category has a discrete feasible space usually made up of regularly arranged points with spacing set to the safety distance between turbines to guarantee solution feasibility. Restricting turbine locations to a finite set of points means it is possible to formulate the layout optimization problem as a mixed integer or binary integer linear program, as demonstrated by Archer *et al.* (2011), Turner *et al.* (2014) and Zhang *et al.* (2014). If the number of feasible points is small (100 points or less), the linear integer program can be solved to optimality using branch and bound algorithms. When problem sizes are too large, or if nonlinear, higher fidelity wind or cost models are used in the objective function, the only practical solution methods are heuristics such as genetic algorithms used by Grady, Hussaini, and Abdullah (2005) and Salcedo-Sanz *et al.* (2013), or greedy algorithms used by Ozturk and Norman (2004), Zhang, Hou, and Wang (2011) and Chen *et al.* (2013).

The second category has a continuous feasible space which, in theory, allows for all possible turbine layouts. However, continuous layout minimization problems from literature are typically non-convex, which means any layout generated by gradient-based optimization methods will be close to the starting layout. Works by Pérez, Mínguez, and Guanche (2013) and Park and Law (2015) have used either a regular or randomly generated starting layout before applying gradient-based optimization techniques to perform fine adjustments of turbine locations. Other works by Kusiak and Song (2010) and Lu and Kim (2014) have paired continuous optimization methods with genetic algorithms to expand the search space, and reduce the solution's dependence on the starting layout. Bio-inspired methods such as particle swarm optimization (Kennedy and Eberhart 1995) have also been used by Chowdhury *et al.* (2013) and Long and Zhang (2015) to increase the chances of finding a globally optimal solution in the continuous feasible space.

The proposed formulation falls in the first category of discrete feasible spaces. Let $\mathbf{x} \in \{0, 1\}^n$ be a vector of binary variables indicating turbine locations among $n$ possible points in the wind farm. There is no requirement that spacing between points must be greater than the safety distance, so dense grids that approximate continuous feasible spaces are possible. The goal of the proposed formulation is to select $t$ feasible turbine locations from $n$ possibilities such that the robustness of the resulting layout is maximized. The directional power output of the wind farm, $F_{t,k\in K,v_0}$, is calculated using the quadratic sum shown in Equation (2). $P(v_0)$ in Equation (2) is the stand-alone power output of a turbine without considering wake effect losses, and $l(v_0)_{k,ij}$ ($l(v_0)_{k,ij} = l(v_0)_{k,ji}$) is the expected power loss caused by turbines placed at points $i$ and $j$ due to the turbines' wakes generated along wind direction $k$ with an incoming wind speed of $v_0$.

$$F_{t,k\in K,v_0}(\mathbf{x}) := \sum_{i=1}^{n} P(v_0)x_i - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} l(v_0)_{k,ij}x_ix_j \tag{2}$$

There are two assumptions built into the definition of $F_{t,k\in K,v_0}$. The first is that the standalone power output of a turbine is independent of its location, which is reasonable if the wind farm is flat, and ambient wind speeds at the site of the wind farm do not vary much with respect to position. The assumption is added for the sake of simplicity, but it is not a strict requirement. The second assumption relates to the calculation of $l(v_0)_{k,ij}$—the wake effect induced power loss caused by turbines placed at points $i$ and $j$. In reality, the expression should be $l(v_0, \mathbf{x})_{k,ij}$ since the incoming wind speed for the upstream turbine among the pair of turbines at $i$ and $j$ depends on the positions of the other turbines in the wind farm. Ignoring the positions of the other turbines means the incoming wind speed to the upstream turbine is always $v_0$. The choice to drop $\mathbf{x}$ from the calculation of mutual power loss reduces the computational cost of evaluating $F_{t,k\in K,v_0}$, making the formulation suitable for cases with a large number of possible locations. In addition, keeping power loss independent of $\mathbf{x}$ retains the quadratic nature of the proposed formulation, which means smaller problem instances could be converted to equivalent Mixed-Integer Linear Programming (MILP) problems and solved using branch and bound.

The proposed formulation for maximizing robustness is expressed as the minimization problem shown in Equation (3). The objective is to minimize $u$, which is the upper bound on the negative directional power outputs across all directions. The second constraint ensures that $t$ locations are selected, and the third constraint prevents the selection of two locations that are separated by less than the safety distance.

$$
\begin{aligned}
\min_{u,x_1,\ldots,x_n} \quad & u \\
\text{subject to :} \quad & u \geq \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} l(v_0)_{k,ij}x_ix_j - \sum_{i=1}^{n} P(v_0)x_i \quad \forall k \in K \\
& \sum_{i=1}^{n} x_i = t \\
& x_i + x_j \leq 1 \quad \forall \text{ infeasible } i < j \\
& x_i \in \{0,1\} \quad \forall i = 1,\ldots,n.
\end{aligned}
\tag{3}
$$

The formulation of Equation (3) can be further simplified by dropping the sum of stand-alone power outputs, since the number of turbines is fixed. The size of $K$ can also be reduced since, in most cases, the set of wind directions is made up of opposite pairs. Let $K_s$ denote the smaller set made up of one direction from every opposing pair. Then the formulation shown in Equation (3) can be expressed in the more compact vector and matrix form shown in Equation (4). The matrix $\mathbf{L}_k(v_0)$ is symmetric for all $k$, with diagonal entries set to 0, and off-diagonal entries $[i,j]$ and $[j,i]$ both set to the value of $l(v_0)_{k,ij}$.

$$
\begin{aligned}
\min_{u,\mathbf{x}} \quad & u \\
\text{subject to :} \quad & u \geq \mathbf{x}^{\mathrm{T}}\mathbf{L}_k(v_0)\mathbf{x} \quad \forall k \in K_s \\
& \mathbf{1}^{\mathrm{T}}\mathbf{x} = t \\
& x_i + x_j \leq 1 \quad \forall \text{ infeasible } i < j \\
& \mathbf{x} \in \{0,1\}^n.
\end{aligned}
\tag{4}
$$

The proposed formulation in Equation (4) reduces to a 0–1 Quadratic Knapsack Problem (QKP) (Pisinger 2007) when there is only a single direction in $K_s$ and there are no infeasible location pairs. The QKP can be converted to an equivalent MILP as seen in Turner *et al.* (2014) and solved using branch and bound for small problem sizes with less than 100 feasible locations. However, finding optimal solutions to QKPs becomes increasingly harder when problem sizes increase since the QKP is NP-hard (Caprara, Pisinger, and Toth 1999). The proposed formulation is likely to be just as difficult as QKPs to solve for large problem sizes, so heuristics such as the greedy algorithm are needed to find good feasible solutions.

## 3. Greedy algorithm with feasibility guarantee

The greedy algorithm has been used successfully in the past to find good solutions to power output maximizing formulations in works by authors such as Zhang, Hou, and Wang (2011) and Song *et al.* (2015). Quan and Kim (2018) showed that the greedy algorithm is able find close-to-optimal solutions to power output maximizing QKPs if the number of turbines and feasible points are not too large. No optimality bounds currently exist for greedy solutions to the proposed formulation, but the proposed formulation's definition of robustness is close in concept to the idea of maximizing the wind farm's power output given a wind profile made up of directions with similar probabilities and wind speeds.

The point-to-point distance in the proposed formulation's feasible space is not required to be greater than the safety distance, so a straightforward application of the greedy algorithm will not always produce feasible solutions. There could be a situation where later iterations of the greedy algorithm run out of feasible points to select. The rest of the section describes a modified greedy algorithm, labelled Greedy-F, that guarantees solution feasibility. An outline of the Greedy-F algorithm is shown in Algorithm 1.

**Algorithm 1** Greedy-F algorithm

Denote $G$ as the set of all possible points, and let $u(X_i)$ be the objective function value with respect to the solution set $X_i$ at iteration $i = 1, \ldots, t$.

Initialize $X_1$ by selecting an arbitrary point, and set $i = 1$.
**while** $i < t$ **do**
  Let $S_i = G \setminus X_i$
  Select $p \in S_i$ such that $u(X_i) - u(X_i \cup p)$ is maximized, subject to:
    Condition 1: $X_i \cup p$ is feasible.
    Condition 2: $f(X_i, C_p) \geq t - i - 1$.
  If such a $p$ exists, add $p$ to $X_i$, and increment $i$ by one. Exit otherwise.
**end while**

The main modification to the basic greedy algorithm is the addition of condition 2 when selecting a new point to add to the solution vector. Condition 2 requires that the number of feasible future selections given by the function $f$ is greater than or equal to the number of turbines that still have to be placed at the end of the current iteration.

The function $f$ takes in two arguments—the current iteration's solution set $X_i$, and the set $C_p$ which contains $p$ and all other points such that the distance between any pair of points in $C_p$ is greater than or equal to the safety distance. $C_p$ can be easily determined in polynomial time for every $p$ before running Greedy-F. The algorithm for constructing $C_p$, labelled getFeasClique is shown in Algorithm 2.

The getFeasClique algorithm starts by populating $C_p$ with all points with distance to $p$ greater than or equal to the safety distance. The algorithm then picks a point in $C_p$ and eliminates all other points from $C_p$ that are closer than the safety distance to the chosen point. It then moves on to the next unpicked point in $C_p$ and repeats the elimination process. The algorithm terminates when there are no more unpicked points in $C_p$, and adds $p$ to $C_p$ in the final step. The end result is a set $C_p$ that forms a feasibility clique, which is a set of points where there is an edge (indicating feasibility) between every pair of points.

**Algorithm 2** getFeasClique algorithm

Let $C_p = \{g \in G \mid d(g,p) \geq s\}$, where $G$ is the set of all points, $d$ is the distance function, and $s$ is the safety distance.
Order the members of $C_p$ arbitrarily, and let $C_p[i]$ denote the $i$th entry in $C_p$.

Set $i = 1$.
**while** $i < |C_p|$ **do**
  $j = i + 1$
  **while** $j \leq |C_p|$ **do**
    **if** $d\left(C_p[i], C_p[j]\right) < s$ **then**
      Remove $C_p[j]$ from $C_p$.
    **else**
      Increment $j$ by one.
    **end if**
  **end while**
  Increment $i$ by one.
**end while**
Add point $p$ to $C_p$.

The function $f$ takes in the sets $X_i$ and $C_p$, and returns the number of points in $C_p$ excluding $p$ that are located greater than or equal to the safety distance from any point in $X_i$. The value returned by function $f$ is the number of all possible feasible turbine locations that can be chosen in future iterations of Greedy-F. Adding this 'look-ahead' feature to the basic greedy algorithm ensures that the Greedy-F algorithm will always have feasible turbine locations to choose from as long as the starting location admits a feasible solution.

The ability of the Greedy-F algorithm to find feasible solutions is demonstrated in Figure 1. The wind farm dimensions in Figure 1 are 1920 by 1920 meters. Possible turbine locations are arranged in a regular square pattern and separated by 160 meters. The Greedy-F algorithm was able to find the only feasible 49-turbine layout shown in Figure 1, whereas the basic greedy algorithm fails at the 33rd iteration.

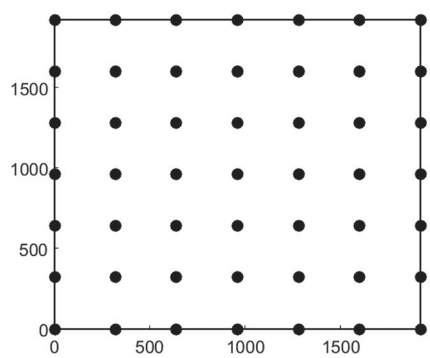


**Figure 1.** Forty-nine-turbine example.

The optimality gap of solutions generated by the Greedy-F algorithm can be determined for small problems. For example, the optimality gaps for a square 1400 by 1400 meter wind farm with 49 uniformly distributed points and 10 turbines are 1.8% and 1.9% based on two wind data sets collected at separate sites in the United States. The 1.8% and 1.9% optimality gaps are equivalent to 9.3% and 13.6% of a single turbine's rated power (1620 kW). The turbine model, wind data, and wake model used in the two examples are the same as those used in the experiments in Section 4. Finding the optimality gaps of much larger problems is highly unlikely, but it could be possible to generate good estimates of the approximate optimality gap provided a tight upper bound on the optimal value of the proposed formulation is available.

## 4. Experiments

This section compares the performance of power output maximizing layouts and robust layouts generated by the Greedy-F algorithm. Improving wind farm robustness will come at the cost of lower total energy generation. The experiments aim to demonstrate with real world data just how large is the trade-off between robustness and energy production. The two approaches were compared over a range of wind farm configurations as shown in Table 1 and Figure 2. Feasible points in the wind farms were distributed throughout the wind farms with a point-to-point distance of around 100 meters, which is the diameter of the turbine discs of the wind turbines used in the experiments. The experiments were conducted twice using wind data from two locations—Storm Lake in Iowa, and Dodge City in Kansas.

The power output maximizing layout was generated by applying the Greedy-F algorithm to the problem shown in Equation (5). The coefficient $e_{ij}$ as defined in Equation (6) represents the expected

**Table 1.** Experiment settings.

| Shape | Size | Turbine counts |
|---|---|---|
| Square, flat, vertical, sheared, circle | Small | 15 (sparse), 20 (dense) |
| | Large | 25 (sparse), 30 (dense) |

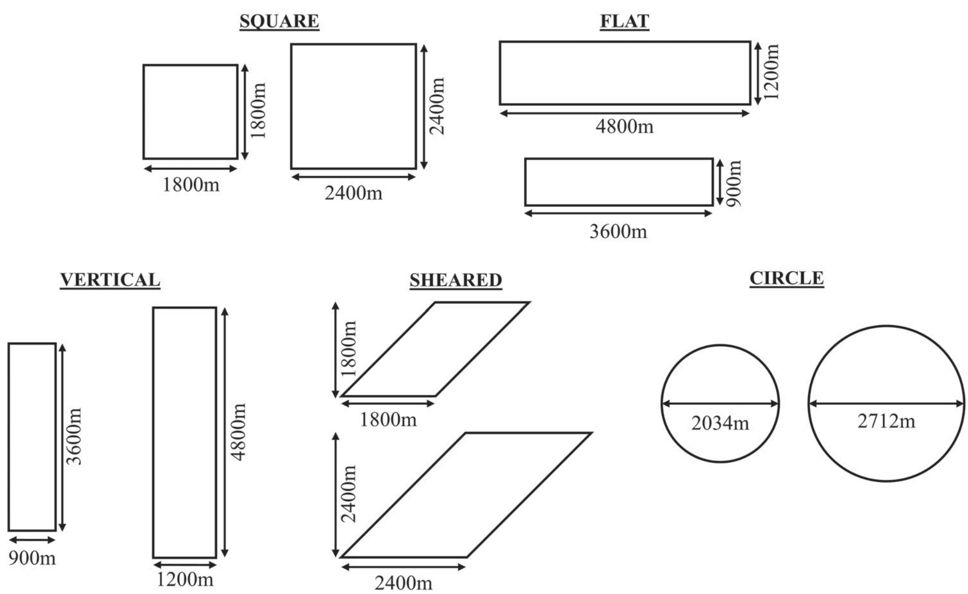


**Figure 2.** Wind farm shapes.

power loss caused by turbines placed at points $i$ and $j$. In Equation (6), $p_k$ represents the probability of wind direction $k$, and $f_k(v)$ is the wind speed distribution for direction $k$ fitted from collected wind data. The term $l(v)_{k,ij}$ in Equation (6) has the same definition as that used in the robust formulation in Equation (3).

$$\begin{aligned} \min_{\mathbf{x}} \quad & \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} e_{ij} x_i x_j \\ \text{subject to :} \quad & \mathbf{1}^{\mathrm{T}} \mathbf{x} = t \\ & x_i + x_j \leq 1 \quad \forall \text{ infeasible } i < j \\ & \mathbf{x} \in \{0, 1\}^n \end{aligned} \tag{5}$$

$$e_{ij} = \sum_{k \in K} \left[ p_k \int_0^{\infty} l(v)_{k,ij} f_k(v) \, \mathrm{d}v \right]. \tag{6}$$

Ten years' worth of hourly wind data collected from two sites in the USA were used for optimization and testing. The wind data were collected by the National Oceanic and Atmospheric Administration (NOAA) at two sites close to actual wind farms—Storm Lake in Iowa, and Dodge City in Kansas. The data collection period was from the years 2005 to 2009, and 2011 to 2015. Wind data from 2010 was not used due to gaps in the data during certain months of the year. The wind speed and direction data for 36 directions were collected at a height of 10 meters. Recorded wind speeds were then extrapolated to turbine hub height using the power law shown in Equation (7), where $v_2$ and $v_1$ are wind speeds at heights $h_2$ and $h_1$, respectively, and $\alpha$ is the wind shear coefficient, which was set to 0.15 as suggested by Patel (1999). Figure 3 shows the distribution of extrapolated wind speeds at Storm Lake and Dodge

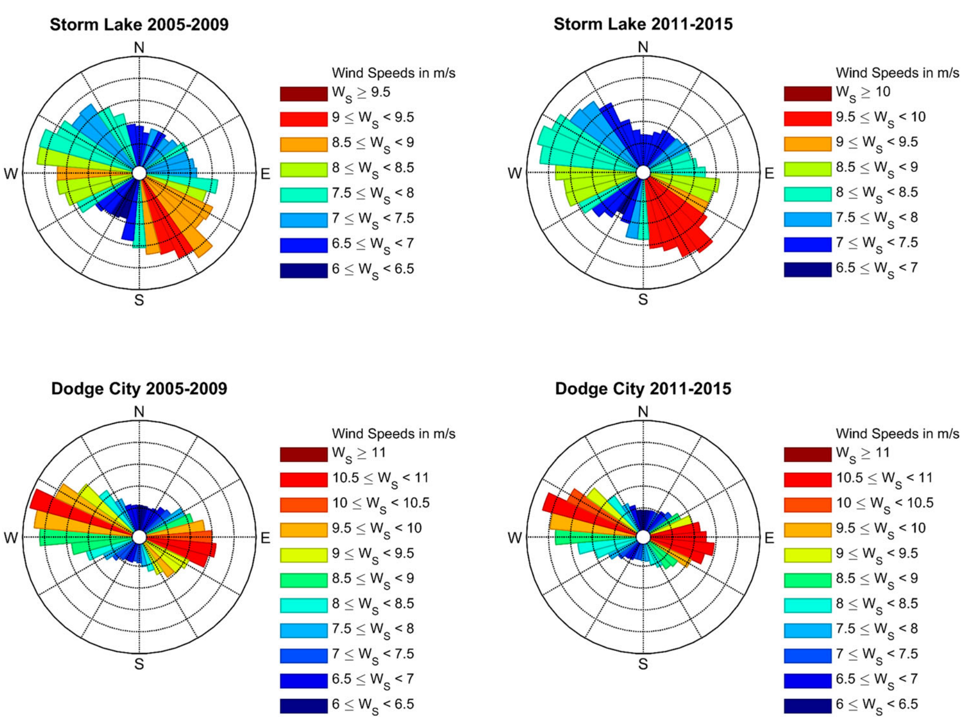


**Figure 3.** Wind roses at Storm Lake and Dodge City.

City for the first five years and the last five years. There are slight variations in the magnitudes of wind speeds, but the dominant directions in both sites show very little drift over the 10 years.

$$v_2 = v_1 \left(\frac{h_2}{h_1}\right)^{\alpha} \tag{7}$$

The first five years of data were used for fitting a Weibull wind speed distribution for each direction, and estimating wind direction probabilities. The wind speed parameter $v_0$ in the robust formulation was set to the average wind speed over the first five years. The last five years of data were used for evaluating layouts in terms of robustness and hourly average power output.

### 4.1. Turbine and wake models

The specifications of the turbine model used at both sites are shown in Table 2. The turbine power and thrust coefficient curves are shown in Figure 4. The turbine specifications and power generation capabilities are based on a turbine produced by General Electric.

The calculation of the power loss coefficient $l(v)_{k,ij}$ in this article was based on the turbine power curve shown in Figure 4 and the wake model developed by Katic, Høstrup, and Jensen (1986). The wake model described in Katic, Høstrup, and Jensen (1986) frequently appears in layout optimization literature, and is typically referred to as the Jensen wake model. The Jensen wake model was selected despite its age due to its good performance when compared to actual wind farms as shown by VanLuvanee (2006), and more recently by Göçmen *et al.* (2016).

The Jensen wake model estimates the incoming wind speeds for a turbine pair located at points $i$ and $j$. Let $v_{k,ij}$ denote the incoming wind speed at location $j$ along direction $k$, given an upstream turbine at point $i$. The expression for $v_{k,ij}$ is given in Equation (8), in which $v$ refers to the ambient wind speed. The term $C_t(v)$ is the turbine's thrust coefficient with respect to $v$, and $D_0$ refers to the diameter of the turbine's disc. The term $A_{k,ij}$ is the area of the turbine's disc at point $j$ that lies inside the wake cone along direction $k$ originating from the turbine at point $i$. If point $j$ is upstream of point $i$ along direction $k$, then $A_{k,ij}$ is zero. The expression $D(d_{k,ij})$ refers to the diameter of the wake cone at a downstream distance of $d_{k,ij}$ along direction $k$ originating from the turbine at point $i$. $D(d_{k,ij})$ is evaluated as shown in Equation (9), where $\kappa$ is the wake expansion coefficient which was set to

**Table 2.** Turbine model specifications.

| | |
|---|---|
| Disc radius | 50 m |
| Hub height | 100 m |
| Power rating | 1.6 MW |
| Rated wind speed | 13 m/s |

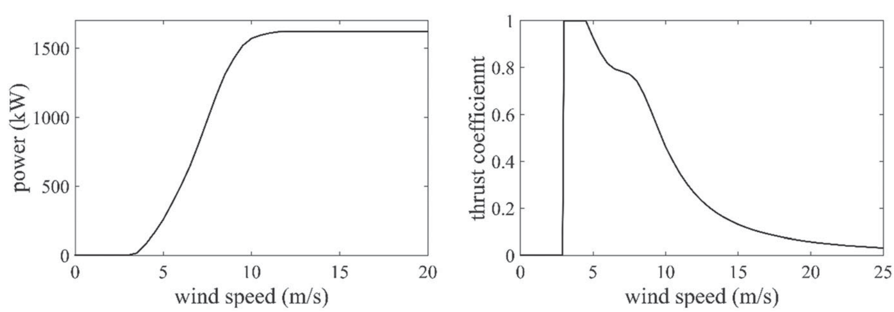


**Figure 4.** Turbine power and thrust coefficient curves.

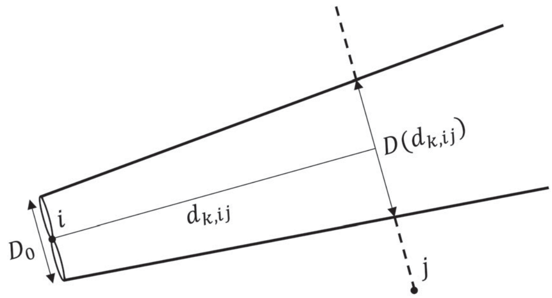


**Figure 5.** Wake cone diameter and downstream distance.

0.075 as recommended by Göçmen *et al.* (2016) for onshore wind farms. The relationship between the diameter of the wake cone $D(d_{k,ij})$ and downstream distance $d_{k,ij}$ is illustrated in Figure 5.

$$v_{k,ij} = \left[1 - \left(\frac{4A_{k,ij}}{\pi}\right)\left(\frac{1-\sqrt{1-C_t(v)}}{D(d_{k,ij})^2}\right)\right] v \tag{8}$$

$$D(d_{k,ij}) = D_0 + 2\kappa\, d_{k,ij}. \tag{9}$$

### *4.2. Results*

Tables 3 and 4 show the minimum power outputs across 72 wind directions (5° separation) of the robust and power output maximizing layouts generated from the Storm Lake and Dodge City data sets. The power output along every direction was calculated using the full Jensen wake model that accounts for wind speed losses from compound wake overlaps using the sum of squares rule. The wind speed parameter $v_0$ used in calculating the directional power outputs was 7.34 m/s for Storm Lake, and 8.32 m/s for Dodge City.

The results in Tables 3 and 4 show that the robust layouts have a higher minimum energy conversion efficiency across all 72 directions compared to the power output maximizing layouts. It is also noteworthy that the differences are larger for non-circular wind farms compared to circular wind farms. The results demonstrate how focusing on power output maximization could result in a layout that has significantly lower energy conversion efficiency along non-dominant wind directions in the collected data.

**Table 3.** Robustness results for Storm Lake.

| | Minimum directional power output (kW) | | | |
|---|---|---|---|---|
| | Sparse | | Dense | |
| Wind farm | Robust | Max. power | Robust | Max. power |
| Small square | 11,759 (+23%) | 9,569 | 14,338 (+12%) | 12,773 |
| Small circle | 11,824 (+2%) | 11,548 | 14,939 (+6%) | 14,099 |
| Small sheared | 11,736 (+13%) | 10,372 | 14,761 (+9%) | 13,557 |
| Small flat | 11,602 (+8%) | 10,738 | 14,215 (+22%) | 11,610 |
| Small vertical | 11,682 (+45%) | 8,049 | 14,405 (+26%) | 11,450 |
| Large square | 19,345 (+10%) | 17,607 | 22,159 (+20%) | 18,424 |
| Large circle | 19,281 (+1%) | 19,158 | 22,619 (+7%) | 21,142 |
| Large sheared | 19,116 (+7%) | 17,823 | 21,658 (+5%) | 20,545 |
| Large flat | 18,955 (+9%) | 17,415 | 21,572 (+13%) | 19,076 |
| Large vertical | 18,955 (+29%) | 14,647 | 21,683 (+41%) | 15,424 |

**Table 4.** Robustness results for Dodge City.

| | Minimum directional power output (kW) | | | |
|---|---|---|---|---|
| | Sparse | | Dense | |
| Wind farm | Robust | Max. power | Robust | Max. power |
| Small square | 16,702 (+20%) | 13,966 | 21,024 (+7%) | 19,680 |
| Small circle | 16,610 (+1%) | 16,365 | 20,994 (+2%) | 20,654 |
| Small sheared | 16,729 (+7%) | 15,641 | 20,790 (+6%) | 19,683 |
| Small flat | 16,441 (+7%) | 15,353 | 20,434 (+15%) | 17,771 |
| Small vertical | 16,778 (+36%) | 12,358 | 21,065 (+32%) | 16,007 |
| Large square | 27,438 (+13%) | 24,200 | 31,906 (+10%) | 29,063 |
| Large circle | 27,583 (+2%) | 26,962 | 32,373 (+4%) | 31,223 |
| Large sheared | 27,493 (+6%) | 25,840 | 31,487 (+6%) | 29,583 |
| Large flat | 27,308 (+8%) | 25,299 | 31,402 (+16%) | 27,094 |
| Large vertical | 27,308 (+42%) | 19,281 | 31,375 (+35%) | 23,174 |

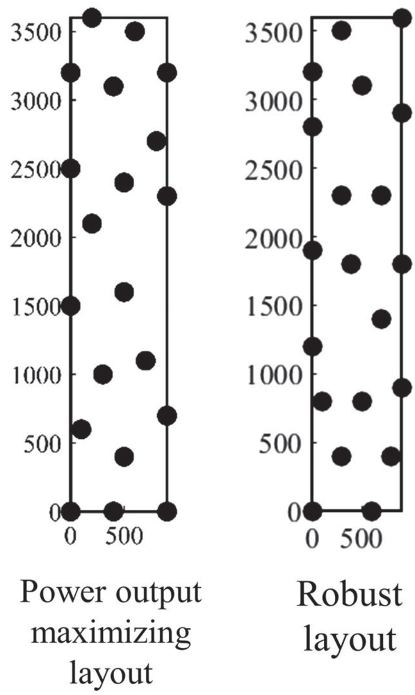


**Figure 6.** Storm Lake vertical wind farm 20-turbine layouts.

A main reason why the robust layouts have much higher minimum directional power outputs is because the average wind speeds at Storm Lake and Dodge City fall within the steepest part of the turbine's power curve between 5 and 10 m/s. This means any reduction in wake overlaps along any direction can lead to noticeable improvements in the wind farm's minimum directional power output. If the average wind speed were to be closer to or higher than the rated wind speed of 10 m/s, then the wind farm's minimum directional power output would be much less sensitive to differences in turbine layout since most turbines in the power output maximizing layout or the robust layout would be operating at around the top flat part of the power curve shown in Figure 4. That is the reason why the definition of robustness in Equation (1) does not just depend on turbine layout, but also $v_0$ which is set to the average wind speed at the site of the wind farm, and not some arbitrary wind speed.

Wind farm shape is another factor that has a big influence on the results. The vertical wind farms which are orientated almost perpendicularly to the dominant wind directions in Storm Lake and Dodge City have the largest differences in minimum directional power output between robust and power output maximizing layouts. Figure 6 shows how the narrowness of the vertical wind farm along the dominant wind direction leads to most turbines being placed along the vertical edges of the wind farm in the power output maximizing layout. As a result, power output along the vertical direction is much lower than what is possible due to the large number of overlapping wake cones. The robust layout, on the other hand, does a much better job at reducing wake overlaps along all directions.

The tendency for power output maximizing layouts to place turbines along the straight edges of a rectangular wind farm in order to create larger separation along dominant wind directions can be avoided when the wind farm is circular. The circular boundary provides a natural way to discourage multiple turbines from lining up along any particular direction as demonstrated in Figure 7, which means the minimum directional power output of power output maximizing layouts do not suffer as much when compared to robust layouts.

Table 5 shows the differences in average hourly power output between the robust and power output maximizing layouts for every wind farm shape and size. The two layout types were compared using the last five years of hourly wind data collected at Storm Lake and Dodge City. It was assumed that wind speeds and directions over the course of an hour did not vary too much from the single data point recorded at the beginning of every hour. The negative values in Table 5 indicate that

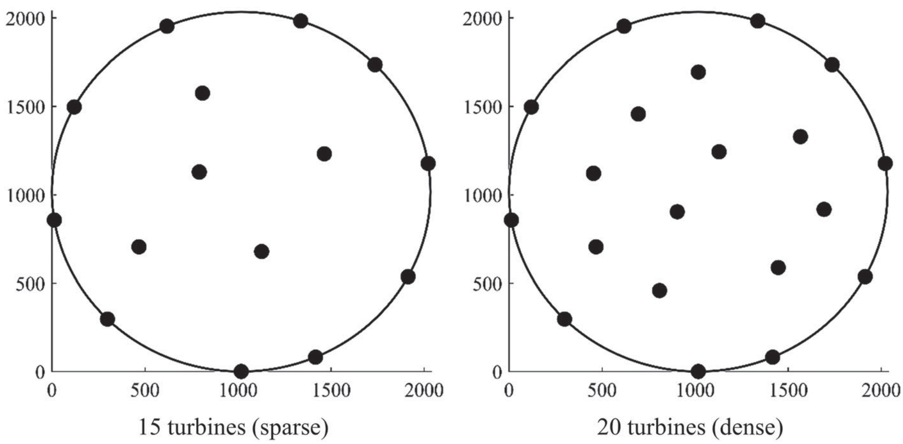


**Figure 7.** Storm Lake circular wind farms with power output maximizing layouts.

**Table 5.** Average hourly power output comparison.

| | Storm Lake | | Dodge City | |
|---|---|---|---|---|
| Wind farm | $\overline{\Delta}$ (kW) | % Δ | $\overline{\Delta}$ (kW) | % Δ |
| Small square | −176 | −1.2% | −209 | −1.3% |
| Small circle | −22 | −0.2% | −91 | −0.6% |
| Small sheared | −163 | −1.1% | −153 | −1.0% |
| Small flat | −193 | −1.3% | −217 | −1.3% |
| Small vertical | −183 | −1.3% | −177 | −1.2% |
| Large square | −264 | −1.2% | −282 | −1.2% |
| Large circle | −200 | −0.9% | −178 | −0.7% |
| Large sheared | −246 | −1.1% | −239 | −1.0% |
| Large flat | −305 | −1.4% | −306 | −1.2% |
| Large vertical | −297 | −1.3% | −367 | −1.5% |

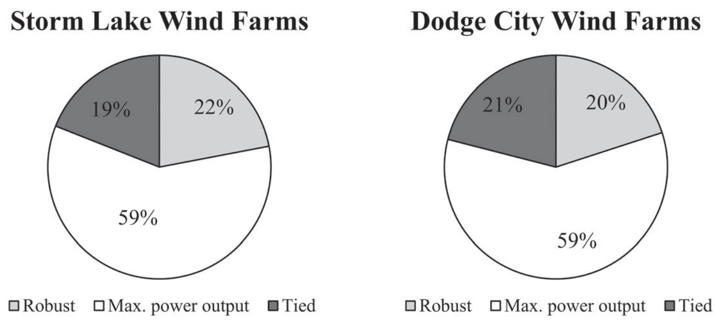


**Figure 8.** Percentage time with highest power output.

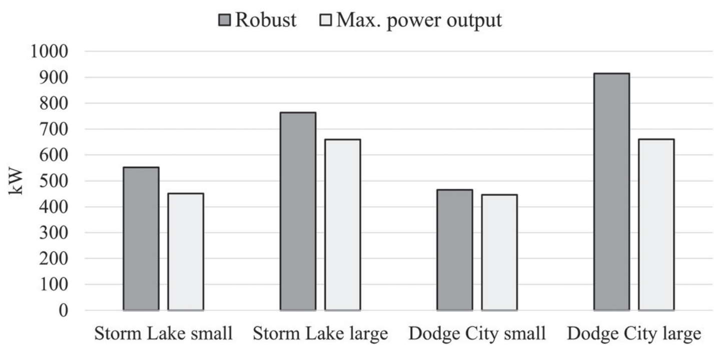


**Figure 9.** Average lead in hourly power output.

the robust layouts are consistently behind power output maximizing layouts in hourly power output. However, the difference is smaller for circular wind farms, which mirrors the robustness results shown in Tables 3 and 4. The average difference is around 1.1% across all wind farm shapes, sizes and turbine counts considered in the experiments. The 1.1% difference represents the 'cost' in energy generation of choosing a robust layout over a power output maximizing layout.

Figure 8 shows the average proportion of time that each layout type had the highest hourly power output. The robust layout was only leading for about 21% of the time, but when it was ahead, its advantage in power output over the power output maximizing layout was higher than when the outcome was reversed. This is reflected in Figure 9, which shows the average leads of the two layout types for various wind farm sizes. The higher average leads of the robust layouts, especially for the larger wind farms, demonstrates the ability of the robust layouts to maintain a high level of power output regardless of wind direction, whereas power output maximizing layouts focus on increasing power output along dominant wind directions at the expense of directions with smaller expected wind speeds.

## 5. Conclusion

This article presented a concept of robust turbine layouts that are characterized by their ability to maintain energy conversion efficiency across all wind directions. A quadratic integer programming formulation was developed for generating robustness maximizing layouts for discrete feasible spaces, along with a modified greedy algorithm (Greedy-F) that can guarantee solution feasibility with regards to inter-turbine safety distance. The proposed method is light on data requirements, only needing an estimate of average wind speeds at the site of the wind farm.

The Greedy-F algorithm is needed for larger problem instances since the proposed formulation is NP-hard. One possible way to reduce the proposed formulation's computational complexity is to impose symmetry on the turbine layout along a certain axis, thereby cutting the number of variables by half. However, this would likely lead to reduced power output along the direction perpendicular to the axis of symmetry since every turbine would have at least another turbine inside its wake cone along that direction. The feasibility of such an approach, and its effectiveness compared to heuristics, could be investigated in the future.

Experiments using wind data collected at two sites showed that the robust layouts traded approximately 1.1% in average hourly power output in return for a higher minimum power output across all wind directions. The decision between the robust and power output maximizing layouts will ultimately depend on the degree of confidence in the wind prediction models, and financial considerations of the wind farm project. The effectiveness of the robust layout, however, depends a great deal on the shape of the wind farm relative to the dominant wind direction. When the wind farm is rectangular and orientated perpendicularly to the dominant wind direction, power output maximizing layouts tend to place turbines along the edges of the wind farm, leading to much lower minimum directional power outputs compared to robust layouts. Circular wind farms can prevent this, and seemed to provide the best balance between power output maximization and robustness compared to rectangular or square wind farms with the same surface area.

Another interesting observation from the experiments is the importance of turbine selection and its impact on robustness. The power output of the wind farm can be maintained across all directions if the rated wind speed of the turbines in the wind farm is close to the average wind speed at the site of the wind farm. However, choosing a turbine model with a lower rated wind speed for the sake of robustness could lead to lower than expected wind farm energy generation. Conversely, if the turbines in the wind farm are operating at less than rated wind speed most of the time, then a robust layout has significantly more stable power output across all directions compared to power output maximizing layouts.

The proposed formulation can be expanded to include turbine selection as a design variable. This can be accomplished by modelling each possible location with a number of points equal to the number of turbine options. The distance between these points can be set to zero to ensure that at most one turbine option is chosen at each location. These modifications would introduce a financial aspect to the problem formulation. A total cost constraint will most likely need to be added to the formulation to ensure that the selected turbines do not exceed a given budget, and the Greedy-F algorithm will need to perform an additional check at every iteration to ensure that there is sufficient budget left for remaining turbines. Exploring the concept of robustness as it applies to the wind farm as a financial investment could be an interesting topic for future work.

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