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# PROBABILISTIC ANALYTICAL TARGET CASCADING – A MOMENT MATCHING FORMULATION FOR MULTILEVEL OPTIMIZATION UNDER UNCERTAINTY

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## ABSTRACT

Analytical target cascading (ATC) is a methodology for hierarchical multilevel system design optimization. In previous work, the deterministic ATC formulation was extended to account for uncertainties using a probabilistic approach. Random quantities were represented by their expected values, which were required to match among subproblems to ensure design consistency. In this work, the probabilistic formulation is augmented to allow introduction and matching of additional probabilistic characteristics. Applying robust design principles, a particular probabilistic analytic target cascading (PATC) formulation is proposed by matching the first two moments of random quantities. Several implementation issues are addressed, including representation of probabilistic design targets, matching interrelated responses and linking variables under uncertainty, and coordination strategies for multilevel optimization. Analytical and simulation-based optimal design examples are used to illustrate the new PATC formulation. Design consistency is achieved by matching the first two moments of interrelated responses and linking variables. The effectiveness of the approach is demonstrated by comparing PATC results to those obtained using a probabilistic all-in-one (PAIO) formulation.

## KEYWORDS

Hierarchical multilevel optimization, analytical target cascading, uncertainty, probabilistic approach, design targets and consistency, coordination strategy, moments

## 1 INTRODUCTION

Optimization of complex systems typically involves a large number of design variables and coupled multidisciplinary analyses. The so-called all-in-one (AIO) approach, in which a large-scale optimization problem is formulated and solved with fully integrated multidisciplinary analyses (MDA), may not be practical as the MDA can be computationally expensive at each optimization iteration. It is often desirable to decompose the system into a number of subsystems each represented by an optimization subproblem. As illustrated in Fig. 1, a system decomposition can be hierarchical (Fig.1a) or nonhierarchical (Fig. 1b).

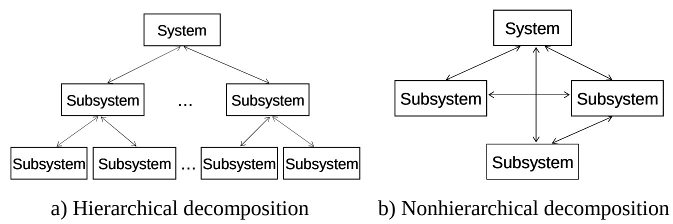


**Figure 1** System decomposition approaches

Multidisciplinary design optimization (MDO) methodologies have been developed to support decomposed, distributed optimization in an effort to maintain disciplinary autonomy under a decentralized, multidisciplinary design environment (Balling and Sobieszczanski-Sobieski, 1996; Alexandrov and Lewis, 1999). Existing MDO techniques (e.g. Cramer *et al.*, 1994; Kroo *et al.*, 1994; Braun *et al.*, 1996; Kroo, 1997; Renaud and Tappeta, 1997) were typically developed for nonhierarchically-decomposed systems. Subsystems are optimized concurrently, while a system-level coordinator is used to take into account subsystem interactions.

Analytical target cascading (ATC) is a methodology developed for hierarchical multilevel system optimization (Kim *et al.*, 2000; Kim, 2001; Kim *et al.*, 2001 and 2002; Kokkolaras *et al.*, 2002 and 2004b). ATC is intended primarily for hierarchies decomposed by objects or physical subsystems rather than by aspects or disciplines (Wagner, 1993), as it is common in MDO. Each block in the hierarchical structure of

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Fig. 1a is referred to as an *element* or a *subproblem*, which can have only one *parent* element, but multiple *children* elements. Unlike many existing MDO formulations, the original problem is decomposed in multiple levels, while interactions among subsystems with the same parent element are considered and coordinated at the level above. ATC operates by pre-specifying system design targets at the top level and formulating and solving a minimum deviation optimization problem[1] for each element in the hierarchy. The system design targets are often determined based on enterprise-level decision-making models (Cooper *et al*., 2003; Wassenaar and Chen, 2003). The process of cascading system targets to design specifications for subsystems at the lower levels of the hierarchy matches the current way of meeting design targets within a hierarchical organizational structure in industry.

MDO formulations including ATC were originally developed for deterministic design problems. Incorporating uncertainty in a MDO formulation is complicated due to the interconnections among multiple elements that exchange information. Efforts to extend MDO to account for uncertainty have been based on integrating either robust design principles (Gu *et al*., 1998; Chen and Lewis, 1999; Du and Chen 2002; Batill *et al*., 2000; Padmanabhan and Batill, 2000; Gurnani and Lewis, 2004) or reliability-based techniques (Sues *et al*., 2000; Chiralaksanakul and Mahadevan, 2004) into MDO formulations. However, most of the research mentioned above is developed for nonhierarchical system optimization problems, which are formulated as single- or bi-level problems.

For design optimization of hierarchically decomposed multilevel systems under uncertainty, Kokkolaras *et al.* (2004a), extended ATC to a probabilistic formulation by using the expected values to represent random quantities communicated among elements. The mean values of interconnected subproblem responses and linking variables are matched, respectively. However, matching the mean values of random quantities may be insufficient to ensure design consistency[2] under uncertainty.

In the present article, we present a more general probabilistic ATC (PATC) formulation that can accommodate various representations of uncertainty in the multilevel optimization. Several issues related to the implementation of the proposed PATC formulation are examined. First, the meaning and representation of design targets under uncertainty are addressed. In our implementation, the quality engineering principle (Phadke, 1989) is followed to set the targets for probabilistic characteristics of engineering attributes throughout the hierarchy. Second, the issue of matching probabilistic behaviors from interrelated elements to ensure design consistency is addressed. It is expected that the degree of matching probabilistic characteristics can have a large impact on the efficiency of the PATC process and the accuracy of the obtained optimal design. In our implementations, design consistency is achieved by matching the first two moments of interrelated responses and linking variables. The effectiveness of this treatment is demonstrated in case studies that compare the results of a probabilistic all-in-one (PAIO) formulation with those from the PATC formulation. Finally, we investigate empirically the potential impact of the coordination strategy on the convergence of PATC by comparing solutions and efficiency of both top-down and bottom-up strategies.

The organization of the article is as follows. In Section 2 we briefly review the deterministic ATC formulation and present a generalized PATC formulation. In Section 3 we take a close look at issues, such as uncertainty representation, design consistency, and coordination strategies. A moment-matching PATC formulation is proposed that matches the mean and standard deviation of random quantities communicated among elements. In Section 4 analytical and simulation-based examples are used to demonstrate the effectiveness of the proposed formulation. Conclusions and suggestions for future work are presented in Section 5.

## 2 PROBABLISTIC ANALYTICAL TARGET CASCADING FORMULATION

### 2.1 Review of the Deterministic Formulation

When viewing engineering product development as a process of meeting targets set by the enterprise level decision-making models (Kim *et al.*, 2004), the deterministic all-in-one (AIO) optimization is formulated as

$$\begin{aligned}&\text{Given} \quad \mathbf{T}\\&\text{find} \quad \mathbf{x}\\&\text{to minimize} \quad \left\|\mathbf{T}-\mathbf{r}(\mathbf{x})\right\|\\&\text{subject to} \quad \mathbf{g}(\mathbf{x})\leq\mathbf{0}.\end{aligned} \tag{1}$$

The vector $\mathbf{x}$ includes all design variables, while the vector $\mathbf{r}$ represents the system's responses. The vector $\mathbf{T}$ includes the target values for $\mathbf{r}$. These target values are fixed during the optimization process. The design objective is to find a feasible design $\mathbf{x}$ that brings the responses $\mathbf{r}$ as close as possible to the assigned targets $\mathbf{T}$. The quality of a design is measured by the deviation between $\mathbf{r}$ and $\mathbf{T}$, using some (possibly weighted) norm. In this paper, we use the $l_2$-norm to measure deviations, but square the norms in the computational implementation of the process to avoid derivative discontinuities.

Using the concept of ATC, the AIO problem in Eq. (1) is decomposed hierarchically into elements at multiple levels. Coupling among elements is captured by linking variables. Linking variables can be design variables shared among elements with the same parent or responses from "sibling" elements at the same level (Allison, 2004). Each element is a subproblem of a smaller size. Inputs to an element include its local design variables, responses from its children elements, and linking variables from "sibling" elements. The design and analysis models at multiple levels are hierarchical by nature as the output of a lower level model becomes the input of a higher level model.

The deterministic ATC optimization of element $j$ at level $i$ ($O_{ij}$) with $n_{ij}$ children is formulated in Eq. (2). The vector $\mathbf{r}_{ij}$ represents the element's responses. The optimization variables include local design variables $\mathbf{x}_{ij}$, linking variables $\mathbf{y}_{ij}$, targets for children responses $\mathbf{r}_{(i+1)k}$, $k=1,\ldots,n_{ij}$, targets for children linking variables $\mathbf{y}_{(i+1)j}$, and tolerance optimization variables $\varepsilon^{\mathrm{r}}$ and $\varepsilon^{\mathrm{y}}$ to coordinate children responses and linking variables for design consistency. The collective optimization variables will from now on be referred to as *decision variables*. Note that

[1] Minimum deviation optimization problem is an optimization problem with the objective to minimize deviations of the achievable performance from the assigned targets.

[2] Design consistency means that the subproblem responses and linking variables are matched among multiple elements, respectively.

element $O_{ij}$ collects all linking variables of its children in a single vector $\mathbf{y}_{(i+1)j}$. The $k^{\text{th}}$ child of $O_{ij}$ uses a selection matrix $\mathbf{S}_{(i+1)k}$, to identify which components of $\mathbf{y}_{(i+1)j}$ correspond to the linking variables $\mathbf{y}_{(i+1)k}$ of that child (Michalek and Papalambros, 2005). Similarly, the $O_{ij}$ itself uses its selection matrix $\mathbf{S}_{ij}$ to identify the target values for its linking variables from the vector $\mathbf{y}_{iq}^{\text{U}}$, where $q$ denotes its parent.

$$
\begin{aligned}
&\text{Given} && \mathbf{r}_{ij}^{\text{U}}, \mathbf{y}_{iq}^{\text{U}}, \mathbf{r}_{(i+1)k}^{\text{L}}, \mathbf{y}_{(i+1)k}^{\text{L}}, \mathbf{S}_{ij}, \mathbf{S}_{(i+1)k}, \quad k=1,\dots,n_{ij}\\
&\text{find} && \mathbf{r}_{(i+1)k}, \mathbf{x}_{ij}, \mathbf{y}_{ij}, \mathbf{y}_{(i+1)j}, \varepsilon_{ij}^{\text{r}}, \varepsilon_{ij}^{\text{y}}, \quad k=1,\dots,n_{ij}\\
&\text{to minimize} && \left\|\mathbf{r}_{ij}-\mathbf{r}_{ij}^{\text{U}}\right\| + \left\|\mathbf{y}_{ij}-\mathbf{S}_{ij}\mathbf{y}_{iq}^{\text{U}}\right\| + \varepsilon_{ij}^{\text{r}} + \varepsilon_{ij}^{\text{y}}\\
&\text{subject to} && \sum_{k=1}^{n_{ij}} \left\|\mathbf{r}_{(i+1)k}-\mathbf{r}_{(i+1)k}^{\text{L}}\right\| \le \varepsilon_{ij}^{\mathbf{r}}\\
& && \sum_{k=1}^{n_{ij}} \left\|\mathbf{S}_{(i+1)k}\mathbf{y}_{(i+1)j}-\mathbf{y}_{(i+1)k}^{\text{L}}\right\| \le \varepsilon_{ij}^{\mathbf{y}}\\
& && \mathbf{g}_{ij}\left(\mathbf{r}_{ij}, \mathbf{x}_{ij}, \mathbf{y}_{ij}\right) \le 0,\\
&\text{where} && \mathbf{r}_{ij} = \mathbf{f}_{ij}\left(\mathbf{r}_{(i+1)1},\dots,\mathbf{r}_{(i+1)n_{ij}}, \mathbf{x}_{ij}, \mathbf{y}_{ij}\right).
\end{aligned}
\tag{2}
$$

In Eq. (2), superscripts U indicate targets assigned by the parent element, while superscripts L indicate values passed from children elements. The targets for responses and linking variables of element $O_{ij}$ are $\mathbf{r}_{ij}^{\text{U}}$ and $\mathbf{S}_{ij}\mathbf{y}_{iq}^{\text{U}}$, respectively. Solving the problem in Eq. (2), element $O_{ij}$ finds the achievable values of its responses and linking variables that are the closest to $\mathbf{r}_{ij}^{\text{U}}$ and $\mathbf{S}_{ij}\mathbf{y}_{iq}^{\text{U}}$, respectively. $O_{ij}$ then passes them back to its parent element as $\mathbf{r}_{ij}^{\text{L}}$ and $\mathbf{y}_{ij}^{\text{L}}$, respectively. It also determines the optimal values for its children responses and linking variables and passes them down as targets, $\mathbf{r}_{(i+1)k}^{\text{U}}$ and $\mathbf{y}_{(i+1)j}^{\text{U}}$. The actual achievable values, $\mathbf{r}_{(i+1)k}^{\text{L}}$ and $\mathbf{y}_{(i+1)k}^{\text{L}}$, are passed up to $O_{ij}$ from its children to maintain consistency.

### 2.2 Generalized Probabilistic ATC Formulation

In a probabilistic design optimization formulation, uncertain quantities are random variables that can be characterized by a probability density function (PDF), a cumulative distribution function (CDF), or descriptors such as moments (Ang and Tang, 1975), etc. We use the superscript ν to denote probabilistic characteristics of a random quantity. For example, for a normally distributed random variable $X$, $\mathbf{X}^{\text{v}}=[\mu_X, \sigma_X]$. Still taking the objective as meeting design targets, the probabilistic AIO (PAIO) optimization formulation is

$$
\begin{aligned}
&\text{Given} && \mathbf{T}^{\text{v}}\\
&\text{find} && \mathbf{X}^{\text{v}}\\
&\text{to minimize} && \left\|\mathbf{T}^{\text{v}}-\mathbf{R}^{\text{v}}\right\|\\
&\text{subject to} && \Pr\left[g_m\left(\mathbf{X}\right)\le 0\right] \ge \alpha_m, \quad m=1,\dots,M,\\
&\text{with} && \mathbf{R}=\mathbf{f}(\mathbf{X}),
\end{aligned}
\tag{3}
$$

where $M$ is the number of constraints. In Eq. (3), capital letters $\mathbf{R}$ and $\mathbf{X}$ are used to represent the random quantities of $\mathbf{r}$ and $\mathbf{x}$ used in Eq. (1). We assume that an appropriate uncertainty propagation technique for computing $\mathbf{R}^{\text{v}}$ is available. Design constraints are posed using the probabilistic feasibility formulation (Du and Chen, 2000), with $\alpha_m$ denoting the required reliability levels. Note that the system design targets vector $\mathbf{T}^{\text{v}}$ in Eq. (3) has a different meaning from $\mathbf{T}$, the targets for deterministic responses in Eq. (1). In the presence of uncertainty, the design target vector $\mathbf{T}^{\text{v}}$ consists of target values that correspond to the probabilistic characteristics $\mathbf{R}^{\text{v}}$. Setting the targets for probabilistic characteristics is important because the variations of system performance can lead to customer dissatisfaction and additional costs to the producer. On the other hand, reducing performance variations often causes increase in the cost of product development. For example, in considering vehicle engine noise under different operating temperatures, design targets should be set for both the nominal value of engine noise and its standard deviation.

Kokkolaras *et al.* (2004a) proposed a PATC formulation, in which expected values (means) are used to represent random variables. For example, in their formulation the characteristic $R^{\text{v}}$ of a random response $R$ is a scalar (the expected value $E(R)$). Accordingly, design targets were only defined for the nominal values of design performance.

In this article, we provide a more general PATC formulation where any interrelated random variables (responses and linking variables) are described by general probabilistic characteristics. The formulation for element $j$ optimization at level $i$ ($O_{ij}$) with $n_{ij}$ children is shown in Eq. (4) (using comma with additional subscript index to denote vector components, e.g., for the constraints).

$$
\begin{aligned}
&\text{Given} && \mathbf{R}_{ij}^{\text{v,U}}, \mathbf{Y}_{iq}^{\text{v,U}}, \mathbf{R}_{(i+1)k}^{\text{v,L}}, \mathbf{Y}_{(i+1)k}^{\text{v,L}}, \mathbf{S}_{ij}, \mathbf{S}_{(i+1)k}, \quad k=1,\dots,n_{ij}\\
&\text{find} && \mathbf{R}_{(i+1)k}^{\text{v}}, \mathbf{X}_{ij}^{\text{v}}, \mathbf{Y}_{ij}^{\text{v}}, \mathbf{Y}_{(i+1)j}^{\text{v}}, \varepsilon_{ij}^{\text{R}}, \varepsilon_{ij}^{\text{Y}}, \quad k=1,\dots,n_{ij}\\
&\text{to minimize} && \left\|\mathbf{R}_{ij}^{\text{v}}-\mathbf{R}_{ij}^{\text{v,U}}\right\| + \left\|\mathbf{Y}_{ij}^{\text{v}}-\mathbf{S}_{ij}\mathbf{Y}_{iq}^{\text{v,U}}\right\| + \varepsilon_{ij}^{\text{R}} + \varepsilon_{ij}^{\text{Y}}\\
&\text{subject to} && \sum_{k=1}^{n_{ij}} \left\|\mathbf{R}_{(i+1)k}^{\text{v}}-\mathbf{R}_{(i+1)k}^{\text{v,L}}\right\| \le \varepsilon_{ij}^{\text{R}},\\
& && \sum_{k=1}^{n_{ij}} \left\|\mathbf{S}_{(i+1)k}\mathbf{Y}_{(i+1)j}^{\text{v}}-\mathbf{Y}_{(i+1)k}^{\text{v,L}}\right\| \le \varepsilon_{ij}^{\text{Y}},\\
& && \Pr[g_{ij,m}\left(\mathbf{R}_{ij}, \mathbf{X}_{ij}, \mathbf{Y}_{ij}\right) \le 0] \ge \alpha_{ij,m}, \quad m=1,\dots,M,\\
&\text{where} && \mathbf{R}_{ij} = \mathbf{f}_{ij}\left(\mathbf{R}_{(i+1)1},\dots,\mathbf{R}_{(i+1)n_{ij}}, \mathbf{X}_{ij}, \mathbf{Y}_{ij}\right).
\end{aligned}
\tag{4}
$$

The above formulation is generally applicable to all the elements of the multilevel hierarchy. Nevertheless, top- and bottom-level problems in PATC are special cases of this formulation. At the top level of the hierarchy ($i$=0), there is only one element $O_0$ (the element index is thus dropped at this level) and there are no linking variables; also, the systems' design targets $\mathbf{R}_0^{\text{v,U}}$ are defined in the vector $\mathbf{T}^{\text{v}}$ in Eq. (3). Elements at the bottom level do not have any children; thus, the first two constraints in Eq. (4) and the $\varepsilon$-variables are eliminated. The structure of this PATC is very similar to the deterministic one in Eq. (2), except that the deterministic targets on responses and linking variables in the objective are now expanded to the targets of probabilistic characteristics of these quantities, and the constraints are expanded to match individual probabilistic characteristics of children responses and linking variables.

## 3 PATC IMPLEMENTATION ISSUES

Three major issues need to be addressed to ensure effective and efficient implementation of the PATC formulation. The first question is what probabilistic characteristics should be used to represent the system level responses and the associated targets. The second issue relates to the choice of probabilistic characteristics to match all interrelated random responses and linking variables for ensuring design consistency under uncertainty. These two issues are discussed in Section 3.1 as they are related to the choice of probabilistic characteristics. This discussion leads to the particular PATC formulation that matches the first two moments of element responses and linking variables, presented in Section 3.2. The third issue, addressed in Section 3.3, relates to choices of coordination strategies for the PATC process given that information regarding uncertainty may be available at different levels in the hierarchy.

### 3.1 Choice of Probabilistic Characteristics

Random variable representation in a PATC formulation depends on the choice of probabilistic characteristics. Moments (e.g., mean and variance) are popular and efficient descriptors of the probabilistic characteristics of random variables. To set up targets for probabilistic characteristics, quality engineering principles can be followed to meet the targets on robustness and reliability of a system, subsystem, or component. The robust design principle is accomplished by bringing the performance mean to its target while reducing the performance variance (Du and Chen, 2000; Kalsi *et al*., 2001; McAllister and Simpson, 2003). Following the robust design principle, targets can be set for the mean and standard deviation of design responses, denoted as $\mathbf{T}^{\mu}$ and $\mathbf{T}^{\sigma}$ for $\boldsymbol{\mu}_{\mathbf{R}}$ and $\boldsymbol{\sigma}_{\mathbf{R}}$ correspondingly, at the top system level. When considering design reliability, targets can be set either for a reliability level $\alpha$ or for a percentile performance corresponding to $\alpha$.

Determining the target values for system level probabilistic characteristics will require introducing an enterprise-driven design approach (Cooper *et al*., 2003; Wassenaar and Chen, 2003; Wassenaar *et al*., 2003 and 2004), which is not the focus of this study. The enterprise decision making model captures the impact of quality engineering characteristics (mean, robustness, reliability, etc.) on product demand and cost, and sets up the targets based on the tradeoffs.

With the PATC, the targets set at the top system level are further cascaded to lower level responses, so as to guide the quality engineering practice throughout the hierarchy. In particular, if targets for mean and standard deviation are set for system level performance, cascading targets on mean and standard deviation throughout the multilevel hierarchy guides robust design efforts at all levels.

In addition to matching assigned targets from a higher level, it is critical to also match the interrelated probabilistic characteristics (responses and linking variables) for ensuring design consistency under uncertainty. Matching the whole distribution is impractical, and the size of the optimization subproblems would also increase substantially if fine discretization is desired. For distributions close to normal, matching only the first two moments is sufficient. Otherwise, higher-order moments may need to be included. In most situations, matching the first four moments would be sufficient but not affordable as the approximation of higher order moments requires additional computational efforts or larger number of samples. Prior knowledge or educated guess of the distribution type is useful for selecting appropriate characteristics to match.

### 3.2 PATC Formulation Based on Matching Mean and Variance

As a particular implementation of the general PATC formulation, we present a PATC formulation that sets the targets on mean and standard deviation for element performance based on robust design considerations and that also matches the interrelated responses and linking variables on the first two moments. The information flow for element $j$ at level $i$ ($O_{ij}$) is shown in Fig. 2.

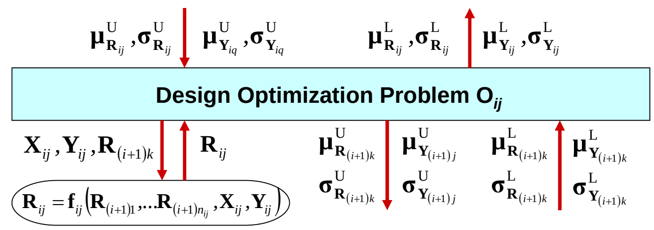



**Figure 2** Information flow for particular PATC formulation

In Fig. 2, $\mathbf{R}_{ij}$ and $\mathbf{Y}_{ij}$ are vectors of random responses and linking variables, respectively. $\mathbf{R}_{ij}$ are evaluated by analysis models $\mathbf{R}_{ij} = \mathbf{f}_{ij}\left(\mathbf{R}_{(i+1)1}, \ldots, \mathbf{R}_{(i+1)n_{ij}}, \mathbf{X}_{ij}, \mathbf{Y}_{ij}\right)$. Targets for the mean and standard deviation of $\mathbf{R}_{ij}$ and $\mathbf{Y}_{ij}$ are assigned by the parent element as [ $\boldsymbol{\mu}^{\mathrm{U}}_{R_{ij}}$ , $\boldsymbol{\sigma}^{\mathrm{U}}_{R_{ij}}$ ] and [ $\boldsymbol{\mu}^{\mathrm{U}}_{Y_{iq}}$ , $\boldsymbol{\sigma}^{\mathrm{U}}_{Y_{iq}}$ ], respectively. Achievable values of mean and standard deviation of $\mathbf{R}_{ij}$ and $\mathbf{Y}_{ij}$ are the output of $O_{ij}$, feeding back to its parent element as [ $\boldsymbol{\mu}^{\mathrm{L}}_{R_{ij}}$ , $\boldsymbol{\sigma}^{\mathrm{L}}_{R_{ij}}$ ] and [ $\boldsymbol{\mu}^{\mathrm{L}}_{Y_{ij}}$ , $\boldsymbol{\sigma}^{\mathrm{L}}_{Y_{ij}}$ ]. Similarly, achievable values of its children element responses and linking variables are passed to $O_{ij}$ as [ $\boldsymbol{\mu}^{\mathrm{L}}_{R_{(i+1)k}}$ , $\boldsymbol{\sigma}^{\mathrm{L}}_{R_{(i+1)k}}$ ] and [ $\boldsymbol{\mu}^{\mathrm{L}}_{Y_{(i+1)k}}$ , $\boldsymbol{\sigma}^{\mathrm{L}}_{Y_{(i+1)k}}$ ], and must be taken into account for consistency. The optimization problem for $O_{ij}$ is solved to find the optimum values for the probabilistic characteristics of its local design variables $\mathbf{X}_{ij}$ and to determine the targets for the responses and linking variables, [ $\boldsymbol{\mu}^{\mathrm{U}}_{R_{(i+1)k}}$ , $\boldsymbol{\sigma}^{\mathrm{U}}_{R_{(i+1)k}}$ ] and [ $\boldsymbol{\mu}^{\mathrm{U}}_{Y_{(i+1)j}}$ , $\boldsymbol{\sigma}^{\mathrm{U}}_{Y_{(i+1)j}}$ ] respectively, of its children elements.

$$
\begin{aligned}
&\text{Given} \quad \boldsymbol{\mu}^{\mathrm{U}}_{\mathbf{R}_{ij}}, \boldsymbol{\sigma}^{\mathrm{U}}_{\mathbf{R}_{ij}}, \boldsymbol{\mu}^{\mathrm{U}}_{\mathbf{Y}_{iq}}, \boldsymbol{\sigma}^{\mathrm{U}}_{\mathbf{Y}_{iq}},\ \boldsymbol{\mu}^{\mathrm{L}}_{\mathbf{R}_{(i+1)k}}, \boldsymbol{\sigma}^{\mathrm{L}}_{\mathbf{R}_{(i+1)k}}, \boldsymbol{\mu}^{\mathrm{L}}_{\mathbf{Y}_{(i+1)k}}, \boldsymbol{\sigma}^{\mathrm{L}}_{\mathbf{Y}_{(i+1)k}}, \mathbf{S}_{ij}, \mathbf{S}_{(i+1)k},\ k = 1,\ldots,n_{ij} \\
&\text{find} \quad \boldsymbol{\mu}_{\mathbf{R}_{(i+1)k}}, \boldsymbol{\sigma}_{\mathbf{R}_{(i+1)k}}, \boldsymbol{\mu}_{\mathbf{X}_{ij}}, \boldsymbol{\sigma}_{\mathbf{X}_{ij}}, \boldsymbol{\mu}_{\mathbf{Y}_{ij}}, \boldsymbol{\sigma}_{\mathbf{Y}_{ij}}, \boldsymbol{\mu}_{\mathbf{Y}_{(i+1)j}}, \boldsymbol{\sigma}_{\mathbf{Y}_{(i+1)j}}, \\
&\qquad \varepsilon^{\mu_{\mathrm{R}}}_{ij}, \varepsilon^{\sigma_{\mathrm{R}}}_{ij}, \varepsilon^{\mu_{\mathrm{Y}}}_{ij}, \varepsilon^{\sigma_{\mathrm{Y}}}_{ij}, \quad k = 1,\ldots,n_{ij} \\
&\text{to minimize} \quad \left\| \boldsymbol{\mu}_{\mathbf{R}_{ij}} - \boldsymbol{\mu}^{\mathrm{U}}_{\mathbf{R}_{ij}} \right\| + \left\| \boldsymbol{\sigma}_{\mathbf{R}_{ij}} - \boldsymbol{\sigma}^{\mathrm{U}}_{\mathbf{R}_{ij}} \right\| + \left\| \boldsymbol{\mu}_{\mathbf{Y}_{ij}} - \mathbf{S}_{ij}\boldsymbol{\mu}^{\mathrm{U}}_{\mathbf{Y}_{iq}} \right\| + \left\| \boldsymbol{\sigma}_{\mathbf{Y}_{ij}} - \mathbf{S}_{ij}\boldsymbol{\sigma}^{\mathrm{U}}_{\mathbf{Y}_{iq}} \right\| \\
&\qquad + \varepsilon^{\mu_{\mathrm{R}}}_{ij} + \varepsilon^{\sigma_{\mathrm{R}}}_{ij} + \varepsilon^{\mu_{\mathrm{Y}}}_{ij} + \varepsilon^{\sigma_{\mathrm{Y}}}_{ij} \\
&\text{subject to} \quad \sum_{k=1}^{n_{ij}} \left\| \boldsymbol{\mu}_{\mathbf{R}_{(i+1)k}} - \boldsymbol{\mu}^{\mathrm{L}}_{\mathbf{R}_{(i+1)k}} \right\| \le \boldsymbol{\varepsilon}^{\mu_{\mathrm{R}}}_{ij}, \qquad \sum_{k=1}^{n_{ij}} \left\| \boldsymbol{\sigma}_{\mathbf{R}_{(i+1)k}} - \boldsymbol{\sigma}^{\mathrm{L}}_{\mathbf{R}_{(i+1)k}} \right\| \le \boldsymbol{\varepsilon}^{\sigma_{\mathrm{R}}}_{ij} \\
&\qquad \sum_{k=1}^{n_{ij}} \left\| \mathbf{S}_{(i+1)k}\boldsymbol{\mu}_{\mathbf{Y}_{(i+1)j}} - \boldsymbol{\mu}^{L}_{\mathbf{Y}_{(i+1)k}} \right\| \le \boldsymbol{\varepsilon}^{\mu_{\mathrm{Y}}}_{ij}, \quad \sum_{k=1}^{n_{ij}} \left\| \mathbf{S}_{(i+1)k}\boldsymbol{\sigma}_{\mathbf{Y}_{(i+1)j}} - \boldsymbol{\sigma}^{L}_{\mathbf{Y}_{(i+1)k}} \right\| \le \boldsymbol{\varepsilon}^{\sigma_{\mathrm{Y}}}_{ij} \\
&\qquad \Pr[g_{ij,m}\left(\mathbf{R}_{ij}, \mathbf{X}_{ij}, \mathbf{Y}_{ij}\right) \le 0] \ge \alpha_{ij,m}, \quad m = 1,\ldots,M, \\
&\text{where} \quad \mathbf{R}_{ij} = \mathbf{f}_{ij}\left(\mathbf{R}_{(i+1)1}, \ldots, \mathbf{R}_{(i+1)n_{ij}}, \mathbf{X}_{ij}, \mathbf{Y}_{ij}\right).
\end{aligned}
\tag{5}
$$

We emphasize that Eq. (5) is a particular PATC formulation. Even though targets and interrelated random quantities are matched for the first two moments in this particular PATC formation, local random variables $\mathbf{X}_{ij}$ are not restricted to normal distributions.

Note that in the above formulation the number of optimization variables is approximately twice as large relative to that of the formulation in Kokkolaras *et al.* (2004a) since each random variable is represented by more than one descriptor, i.e., the first two moments.

### 3.3 Coordination Strategies

Similar to the deterministic ATC, the PATC is an iterative process based on the optimization of subproblems until deviations of probabilistic system responses from targets cannot be reduced any further without violating system consistency. This iterative process requires an appropriate coordination strategy to ensure convergence. Michelena *et al.* (2002) proved convergence properties of the deterministic ATC formulation for a specific class of coordination strategies under standard convexity and smoothness assumptions.

When dealing with uncertainties that propagate throughout the multilevel hierarchy, one question is at which level the PATC process should begin. From an organization's viewpoint, the design process should start from the highest level, as usually overall targets are assigned and cascaded down from top to bottom. On the other hand, it may be beneficial to start at the level where uncertainty cannot be reduced, i.e., at the level where we cannot control the variation of random inputs. Typically, this happens at the bottom level, where most random design variables have known distributions. The bottom-up coordination strategy imitates the uncertainty propagation process. In this study, both strategies are tested to investigate whether the starting level has an impact on convergence speed and solution accuracy. Note that we have not conducted a theoretical study of convergence properties.

## 4 EXAMPLES

The first example is the geometric programming problem adopted from Kim *et al.* (2000). The second example is the design of the V6 gasoline engine in Kokkolaras *et al.* (2004a). The examples are used to investigate whether matching the first two moments is sufficient, by comparing results from PATC with those from PAIO. A preliminary investigation on coordination strategies, top-down and bottom-up, is also conducted.

### 4.1 Geometric Programming Problem

#### 4.1.1 Problem Formulation

Geometric programming problems with posynomials (polynomials with positive constants) are known to have a unique global optimum (Beightler and Phillips, 1976). The deterministic AIO and ATC formulations of the geometric programming problem are provided in Kim *et al.* (2000). The PAIO problem is formulated in Eq. (6), and the purpose of solving it is to verify whether the PATC is capable of reaching the reaching the same optimal solution. In Eq. (6), capital letters are used to represent random quantities, while lower cases are kept for deterministic quantities or realizations of random quantities.

$$
\begin{aligned}
&\text{Given} && T^{\mu_{X_1}},\ T^{\sigma_{X_1}}, T^{\mu_{X_2}}, T^{\sigma_{X_2}}, \sigma_{X_8}, \sigma_{X_{11}} \\
&\text{find} && x_4, x_5, x_7, \mu_{X_8}, x_9, x_{10}, \mu_{X_{11}}, x_{12}, x_{13}, x_{14} \ge 0 \\
&\text{to minimize} && \left(T^{\mu_{X_1}} - \mu_{X_1}\right)^2 + \left(T^{\sigma_{X_1}} - \sigma_{X_1}\right)^2 + \left(T^{\mu_{X_2}} - \mu_{X_2}\right)^2 + \left(T^{\sigma_{X_2}} - \sigma_{X_2}\right)^2 \\
&\text{subject to} && \Pr\left(g_i \le 0\right) \ge \alpha_i, \quad i = 1, \dots, 6 \\
&\text{where} && X_1 = \left(X_3^2 + x_4^{-2} + x_5^2\right)^{1/2} \qquad X_2 = \left(x_5^2 + X_6^2 + x_7^2\right)^{1/2} \\
& && X_3 = \left(X_8^2 + x_9^{-2} + x_{10}^{-2} + X_{11}^2\right)^{1/2} \qquad X_6 = \left(X_{11}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2\right)^{1/2} \\
& && g_1 = \frac{X_3^{-2} + x_4^2}{x_5^2} - 1, \quad g_2 = \frac{x_5^2 + X_6^{-2}}{x_7^2} - 1, \quad g_3 = \frac{X_8^2 + x_9^{-2}}{X_{11}^2} - 1, \\
& && g_4 = \frac{X_8^{-2} + x_{10}^2}{X_{11}^2} - 1, \quad g_5 = \frac{X_{11}^2 + x_{12}^{-2}}{x_{13}^2} - 1, \quad g_6 = \frac{X_{11}^2 + x_{12}^2}{x_{14}^2} - 1.
\end{aligned}
\tag{6}
$$

The functional dependencies in Eq. (6) are used to decompose the problem into a bi-level hierarchy with two elements at the bottom level. Setting $R_{0,1}=X_1$, $R_{0,2}=X_2$, $\mathbf{x}_0=[x_4,x_5,x_7]$, $R_{11}=X_3$, and $R_{12}=X_6$, the top-level responses are computed by

$$
\mathbf{R}_0 = \begin{bmatrix} R_{0,1} \\ R_{0,2} \end{bmatrix} = \mathbf{f}_0(R_{11}, R_{12}, \mathbf{x}_0) = \begin{bmatrix} \left(R_{11}^2 + x_4^{-2} + x_5^2\right)^{1/2} \\ \left(x_5^2 + R_{12}^2 + x_7^2\right)^{1/2} \end{bmatrix},
\tag{7}
$$

where a comma and additional index denote vector component. Similarly, setting $\mathbf{X}_{11}=[X_8,x_9,x_{10}]$, $Y_{11}=X_{11}$, $\mathbf{x}_{12}=[x_{12},x_{13},x_{14}]$, and $Y_{12}=X_{11}$, the two bottom-level element responses are computed by

$$
R_{11} = f_{11}(\mathbf{X}_{11}, Y_{11}) = \left(X_8^2 + x_9^{-2} + x_{10}^{-2} + X_{11}^2\right)^{1/2},
\tag{8}
$$

and

$$
R_{12} = f_{12}(\mathbf{x}_{12}, Y_{12}) = \left(X_{11}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2\right)^{1/2} \quad .
\tag{9}
$$

The bi-level decomposition of the PAIO problem and the associated information flow are shown in Fig. 3.

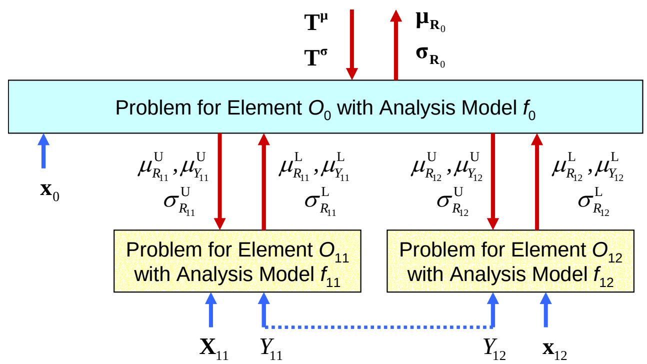



**Figure 3** Information flow in the bi-level hierarchical decomposition of geometric programming problem

We assume that design variables $X_8$ and $X_{11}$ are independent and normally distributed with constant standard deviations $\sigma_{X_8} = \sigma_{X_{11}}$ =0.1. The randomness in $X_8$ and $X_{11}$ results in uncertainties in all computed responses, each described by its mean and standard deviation. Overall system targets are given as $[\mathbf{T}^{\mu}, \mathbf{T}^{\sigma}]$. Since elements $O_{11}$ and $O_{12}$ share the random design variable $X_{11}$, it becomes a random linking design variable, i.e., $Y_{11}=X_{11}$ and $Y_{12}=X_{11}$.

The primary goal of this example is to test the effectiveness of the proposed particular PATC method. We use Monte Carlo Simulation (MCS) to evaluate the first two moments of responses to avoid the influence caused by approximation methods for the mean and variance estimation. All probabilistic constraints are evaluated by the moment-matching method:

$$
\mu_g + k\sigma_g \le 0 \,,
\tag{10}
$$

where $\mu_g$ and $\sigma_g$ are also obtained by MCS. The probabilistic optimization models for the three elements $O_0$, $O_{11}$, and $O_{12}$ in Fig. 3 under the particular PATC formulation are formulated in Eqs. (11)-(13), respectively. Note that since the standard deviation of the random variable $X_{11}$ is assumed constant (i.e., cannot be controlled), it is not included as an optimization variable. In general, if we cannot control the standard deviation of a random response or a linking variable, we are forced to omit it from the particular moment-matching formulation of Eq. (5).

$$
\begin{aligned}
&O_0: \text{Given} \quad \mathbf{T}^{\boldsymbol{\mu}},\ \mathbf{T}^{\boldsymbol{\sigma}},\ \mu_{R_{11}}^L,\ \sigma_{R_{11}}^L,\ \mu_{R_{12}}^L,\ \sigma_{R_{12}}^L,\ \mu_{Y_{11}}^L,\ \mu_{Y_{12}}^L \\
&\text{find} \quad x_4,\ x_5,\ x_7,\ \mu_{R_{11}},\ \sigma_{R_{11}},\ \mu_{R_{12}},\ \sigma_{R_{12}},\ \mu_{Y_1},\ \varepsilon^{\mu_R},\ \varepsilon^{\sigma_R},\ \varepsilon^{\mu_Y} \ge 0 \\
&\text{to minimize} \quad \left\| \mathbf{T}^{\boldsymbol{\mu}} - \boldsymbol{\mu}_{\mathbf{R}_0} \right\|_2^2 + \left\| \mathbf{T}^{\boldsymbol{\sigma}} - \boldsymbol{\sigma}_{\mathbf{R}_0} \right\|_2^2 + \varepsilon^{\mu_R} + \varepsilon^{\sigma_R} + \varepsilon^{\mu_Y} \\
&\text{subject to} \quad \left(\mu_{R_{11}} - \mu_{R_{11}}^L\right)^2 + \left(\mu_{R_{12}} - \mu_{R_{12}}^L\right)^2 \le \varepsilon^{\mu_R}, \\
&\qquad \left(\sigma_{R_{11}} - \sigma_{R_{11}}^L\right)^2 + \left(\sigma_{R_{12}} - \sigma_{R_{12}}^L\right)^2 \le \varepsilon^{\sigma_R}, \\
&\qquad \left(\mu_{Y_1} - \mu_{Y_{11}}^L\right)^2 + \left(\mu_{Y_1} - \mu_{Y_{12}}^L\right)^2 \le \varepsilon^{\mu_Y}, \\
&\qquad \mu_{g_1} + 3\sigma_{g_1} \le 0, \qquad \mu_{g_2} + 3\sigma_{g_2} \le 0, \\
&\text{where} \quad \mathbf{R}_0 = \begin{bmatrix} R_{0,1} \\ R_{0,2} \end{bmatrix} = \begin{bmatrix} \left(R_{11}^2 + x_4^{-2} + x_5^2\right)^{1/2} \\ \left(x_5^2 + R_{12}^2 + x_7^2\right)^{1/2} \end{bmatrix} \\
&\qquad g_1 = \frac{R_{11}^{-2} + x_4^2}{x_5^2} - 1, \quad g_2 = \frac{x_5^2 + R_{12}^{-2}}{x_7^2} - 1.
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
&O_{11}: \text{Given} \quad \mu_{R_{11}}^U, \sigma_{R_{11}}^U, \mu_{Y_1}^U, \sigma_{Y_1}^U, \sigma_{X_8}, \sigma_{Y_{11}} (= \sigma_{X_{11}}) \\
&\text{find} \quad \mu_{X_8}, x_9, x_{10}, \mu_{Y_{11}} \ge 0 \\
&\text{to minimize} \quad \left(\mu_{R_{11}} - \mu_{R_{11}}^U\right)^2 + \left(\sigma_{R_{11}} - \sigma_{R_{11}}^U\right)^2 + \left(\mu_{Y_{11}} - \mu_{Y_1}^U\right)^2 \\
&\text{subject to} \quad \mu_{g_3} + 3\sigma_{g_3} \le 0 \\
&\qquad \mu_{g_4} + 3\sigma_{g_4} \le 0, \\
&\text{where} \quad R_{11} = \left(X_8^2 + x_9^{-2} + x_{10}^{-2} + Y_{11}^2\right)^{1/2} \\
&\qquad g_3 = \frac{X_8^2 + x_9^{-2}}{Y_{11}^2} - 1, \quad g_4 = \frac{X_8^{-2} + x_{10}^2}{Y_{11}^2} - 1.
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
&O_{12}: \text{Given} \quad \mu_{R_{12}}^U, \sigma_{R_{12}}^U, \mu_{Y_1}^U, \sigma_{Y_1}^U, \sigma_{Y_{12}} (= \sigma_{X_{11}}) \\
&\text{find} \quad \mu_{Y_{12}}, x_{12}, x_{13}, x_{14} \ge 0 \\
&\text{to minimize} \quad \left(\mu_{R_{12}} - \mu_{R_{12}}^U\right)^2 + \left(\sigma_{R_{12}} - \sigma_{R_{12}}^U\right)^2 + \left(\mu_{Y_{12}} - \mu_{Y_1}^U\right)^2 \\
&\text{subject to} \quad \mu_{g_5} + 3\sigma_{g_5} \le 0 \\
&\qquad \mu_{g_6} + 3\sigma_{g_6} \le 0, \\
&\text{where} \quad R_{12} = \left(Y_{12}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2\right)^{1/2} \\
&\qquad g_5 = \frac{Y_{12}^2 + x_{12}^{-2}}{x_{13}^2} - 1, \quad g_6 = \frac{Y_{12}^2 + x_{12}^2}{x_{14}^2} - 1.
\end{aligned} \tag{13}
$$

Both top-down and bottom-up coordination strategies were tested. Starting from the top level requires an initial guess of $\left[\mu_{Y_{11}}^U, \mu_{Y_{12}}^U\right]$ and $\left[\sigma_{Y_{11}}^U, \sigma_{Y_{12}}^U\right]$ when solving $O_{11}$, and $O_{12}$ for the first time. Starting from the bottom level required an initial guess of $\left[\mu_{R_{11}}^U, \mu_{R_{12}}^U\right]$ and $\left[\sigma_{R_{11}}^U, \sigma_{R_{12}}^U\right]$ when solving $O_0$ for the first time. For this example, the obtained optimal solutions were identical under both coordination strategies. The completion of optimizations in all elements is considered as one PATC *cycle*.

### 4.1.2 PATC Results

For the results presented here, the target values for the mean and the standard deviation of the system response $R_0$ were $\mathbf{T}^{\boldsymbol{\mu}}$ = [0, 0] and $\mathbf{T}^{\boldsymbol{\sigma}}$ = [0, 0]. For each MCS, 10,000 samples were used. When the maximum value of deviation terms on $\varepsilon$ in $O_0$ was within allowable tolerance (1.0*e*-4), and when each element optimization converged successfully, the whole PATC process was considered to have converged to an optimal design. For the specified tolerance of consistency (1.0*e*-4), 136 cycles were used to reach the convergence for both top-down and bottom-up strategies. The optimal design and system-level performances obtained by starting the PATC from the top level are listed in Tables 1 and 2, respectively. Table 3 compares targets assigned by $O_0$ for the mean of the linking variable and the actual values obtained by $O_{11}$ and $O_{12}$.

**Table 1** Comparison of optimal designs

| | Initial Point | PAIO | PATC |
|---|---|---|---|
| $x_4$ | 5.0 | 0.7599 | 0.7597 |
| $x_5$ | 5.0 | 0.8676 | 0.8659 |
| $x_7$ | 5.0 | 0.9208 | 0.9209 |
| $\mu_{X_8}$ | 5.0 | 1.1984 | 1.2013 |
| $x_9$ | 5.0 | 0.8098 | 0.7912 |
| $x_{10}$ | 5.0 | 0.7350 | 0.7229 |
| $\mu_{X_{11}}$ | 5.0 | 1.4931 | 1.4737 |
| $x_{12}$ | 5.0 | 0.8409 | 0.8419 |
| $x_{13}$ | 5.0 | 2.1333 | 2.1080 |
| $x_{14}$ | 5.0 | 1.9606 | 1.9344 |

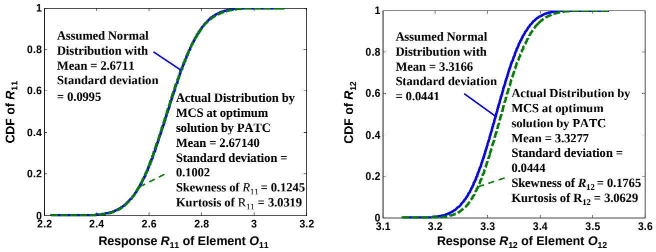
**Table 2** Comparison of optimal solutions

| | PAIO | PATC | Confirmed PATC Solution |
|---|---|---|---|
| $\left[\mu_{R_{0,1}}^*, \mu_{R_{0,2}}^*\right]$ | [3.0875, 3.5968] | [3.1019, 3.5599] | [3.1006, 3.5488] |
| $\left[\sigma_{R_{0,1}}^*, \sigma_{R_{0,2}}^*\right]$ | [0.0874, 0.0417] | [0.0862, 0.0414] | [0.0860, 0.0413] |
| Objective function | 22.4790 | 22.3038 | 22.2168 |

**Table 3** Comparison of linking variable mean values

| | Target Value by $O_0$ | Actual Value at $O_{11}$ | Actual Value at $O_{12}$ |
|---|---|---|---|
| $\mu_Y$ | 1.4735 | 1.4834 | 1.4640 |

Tables 1 and 2 show that PATC converges to the same optimal solution as that obtained by PAIO. Table 3 shows that the optimal mean value of the shared design variable of the two coupled elements $O_{11}$ and $O_{12}$ is consistent. Since only the first two moments were matched during the PATC process, the optimal solution was verified by substituting the optimal design point back into fully integrated analysis models in PAIO and computing the true values of $\left[\mu_{R_{0,1}}^*, \mu_{R_{0,2}}^*\right]$ and $\left[\sigma_{R_{0,1}}^*, \sigma_{R_{0,2}}^*\right]$. The results are listed in the last column in Table 2. They are sufficiently close to those from PATC, indicating that the use of the first two moments for matching probabilistic behaviors is sufficient for this example. True distributions of $R_{11}$ and $R_{12}$ obtained from MCS using 100,000 samples are compared to those incorporating the first two moments only (i.e., assuming normal distributions in $O_0$) in Fig. 4. The comparison further illustrates that matching the first two moments in PATC is sufficient in this case as the lower level element responses $R_{11}$ and $R_{12}$ follow distributions close to normal distributions and the linking variable $X_{11}$ is also normally distributed.




a) Verification of distribution of $R_{11}$ b) Verification of distribution of $R_{12}$

**Figure 4** Comparison of actual CDFs of responses of $O_{11}$ and $O_{12}$ with the assumed normal distribution CDFs in $O_0$

PATC reached the same optimum when starting from different initial points. We tested different values of weighting factors for the terms of the objective function in the problem formulation for element $O_0$. If weighing factors for the consistency terms ($\varepsilon$) are too large, e.g., 1000, the PATC formulation quickly converges to a consistent but suboptimal solution. With constant weighting factors, we observed that PATC converges to the global optimum, but many cycles are needed to fine-tune the search so as to meet the consistency of children elements optimization.

Although not shown explicitly in the formulations, weighting factors are introduced to capture tradeoffs among different deviation terms in the objective function and consistency constraints. A heuristic adaptive weighting scheme that increases the values of weighting factors for the deviation terms on $\varepsilon$ with the increase of PATC cycles was used. A formal method for setting proper weights for element responses and linking variables in deterministic ATC can be found in Michalek *et al.* (2005).

## 4.2 Piston-Ring/Cylinder-Liner Design Problem

### 4.2.1 Problem Formulation

To investigate the validity of moment-matching when element responses are known to be not normally distributed, and to investigate the performance of top-down and bottom-up coordination strategies, we use the same example as in Kokkolaras *et al.* (2004a). The piston-ring/cylinder-liner subassembly is designed to minimize fuel consumption of a V6 engine while satisfying reliability requirements on oil consumption, blow-by, and liner wear rate. A target reliability level is chosen as 99.87%, corresponding to a reliability index as 3. The PAIO problem formulation is given in Eq. (14).

$$
\begin{aligned}
&\text{Given} \quad T^{\mu}, T^{\sigma}, \sigma_{X_1}, \sigma_{X_2}\\
&\text{find} \quad \mu_{X_1}, \mu_{X_2}, x_3, x_4\\
&\text{to minimize} \quad \left(T^{\mu}-\mu_{R_{fuel}}\right)^2+\left(T^{\sigma}-\sigma_{R_{fuel}}\right)^2\\
&\text{subject to} \quad \Pr\left(R_{\text{wear}} \le 2.4\times10^{-12}\ \ \mathrm{m^3/s}\right) \ge 99.87\%\\
&\qquad\qquad \Pr\left(R_{\text{blow by}} \le 4.25\times10^{-5}\ \ \mathrm{kg/s}\right) \ge 99.87\%\\
&\qquad\qquad \Pr\left(R_{\text{oil}} \le 15.3\ \ \mathrm{g/hr}\right) \ge 99.87\%\\
&\qquad\qquad 4\ \mu\mathrm{m} \le \mu_{X_1} \le 7\ \mu\mathrm{m},\ \ 4\ \mu\mathrm{m} \le \mu_{X_2} \le 7\ \mu\mathrm{m}\\
&\qquad\qquad 80\ \mathrm{GPa} \le x_3 \le 340\ \mathrm{GPa}, \quad 150\ \mathrm{BHV} \le x_4 \le 240\ \mathrm{BHV}\\
&\text{where} \quad R_{\text{fuel}} = f_{\text{fuel}}\left(R_{\text{power loss}}\right), \quad R_{\text{power loss}} = f_{\text{power loss}}\left(X_1, X_2, x_3, x_4\right),\\
&\qquad R_{\text{wear}} = f_{\text{wear}}\left(X_1, X_2, x_3, x_4\right), \quad R_{\text{blow by}} = f_{\text{blow by}}\left(X_1, X_2, x_3, x_4\right),\\
&\qquad R_{\text{oil}} = f_{\text{oil}}\left(X_1, X_2, x_3, x_4\right).
\end{aligned}
\tag{14}
$$

The problem is decomposed into a two-level hierarchy with only one element at each level. The four design variables are inputs to the bottom-level element, whose responses $\boldsymbol{R}_1$ include power loss due to ring/liner friction ($R_{\text{power loss}}$), liner wear rate ($R_{\text{wear}}$), blow-by ($R_{\text{blow by}}$), and oil consumption ($R_{\text{oil}}$). The top-level element takes only the power loss response $R_{1,1}$ (= $R_{\text{power loss}}$) as input and provides fuel consumption as the system level response $R_0$ (= $R_{\text{fuel}}$). Once again, a comma and additional subscript index denotes a vector component.

Following the PATC formulation presented in Section 3.2, the mean and standard deviation are selected as the probabilistic characteristics for responses. The top level problem $O_0$ is formulated in Eq. (15). Because there is only one element at each level, there are no linking variables in this example. PATC constraints in $O_0$ are used to ensure consistency of child element responses with obtained targets.

$$
\begin{aligned}
&O_0: \text{Given} \quad T^{\mu}, T^{\sigma}, \mu^{L}_{R_{1,1}}, \sigma^{L}_{R_{1,1}}\\
&\text{find} \quad \mu_{R_{1,1}}, \sigma_{R_{1,1}}, \varepsilon^{\mu}, \varepsilon^{\sigma}\\
&\text{to minimize} \quad \left(T^{\mu}-\mu_{R_0}\right)^2+\left(T^{\sigma}-\sigma_{R_0}\right)^2+\varepsilon^{\mu}+\varepsilon^{\sigma}\\
&\text{subject to} \quad \left(\mu_{R_{1,1}}-\mu^{L}_{R_{1,1}}\right)^2 \le \varepsilon^{\mu}\\
&\qquad\qquad \left(\sigma_{R_{1,1}}-\sigma^{L}_{R_{1,1}}\right)^2 \le \varepsilon^{\sigma},\\
&\text{where} \quad R_0 = R_{fuel} = f_0\left(R_{1,1}\right).
\end{aligned}
\tag{15}
$$

The formulation of the bottom level element $O_1$ according to the PATC process is shown in Eq. (16). The random design variables $X_1$ and $X_2$ denote piston-ring and cylinder-liner surface roughness, respectively. Based on measurements, they are normally distributed with $\sigma_{X_1}=\sigma_{X_2}=\ 1\mu m$. The deterministic design variables $x_3$ and $x_4$ denote Young's modulus and hardness of the liner material, respectively. There are three reliability constraints related to the subassembly's performance, i.e., liner wear rate ($R_{\text{wear}}$), blow-by ($R_{\text{blow by}}$), and oil consumption ($R_{\text{oil}}$). The problem for element $O_1$ is solved by the Sequential Optimization and Reliability Assessment (SORA) method (Du and Chen, 2004). To ease the computational burden, the analysis models in $O_0$ and $O_1$ are surrogate models built using the cross-validated moving least squares method (Kokkolaras *et al.*, 2004a).

$$
\begin{aligned}
&O_1: \text{Given} \quad \mu^{U}_{R_{1,1}}, \sigma^{U}_{R_{1,1}}, \sigma_{X_1}, \sigma_{X_2}\\
&\text{find} \quad \mu_{X_1}, \mu_{X_2}, x_3, x_4\\
&\text{to minimize} \quad \left(\mu_{R_{1,1}}-\mu^{U}_{R_{1,1}}\right)^2+\left(\sigma_{R_{1,1}}-\sigma^{U}_{R_{1,1}}\right)^2\\
&\text{subject to} \quad \Pr\left(R_{\text{wear}} \le 2.4\times10^{-12}\ \ \mathrm{m^3/s}\right) \ge 99.87\%\\
&\qquad\qquad \Pr\left(R_{\text{blow by}} \le 4.25\times10^{-5}\ \ \mathrm{kg/s}\right) \ge 99.87\%\\
&\qquad\qquad \Pr\left(R_{\text{oil}} \le 15.3\ \ \mathrm{g/hr}\right) \ge 99.87\%\\
&\qquad\qquad 4\ \mu\mathrm{m} \le \mu_{X_1} \le 7\ \mu\mathrm{m},\ \ 4\ \mu\mathrm{m} \le \mu_{X_2} \le 7\ \mu\mathrm{m}\\
&\qquad\qquad 80\ \mathrm{GPa} \le x_3 \le 340\ \mathrm{GPa}, \quad 150\ \mathrm{BHV} \le x_4 \le 240\ \mathrm{BHV},\\
&\text{where} \quad \mathbf{R}_1 = \begin{bmatrix} R_{1,1}\\ R_{1,2}\\ R_{1,3}\\ R_{1,4} \end{bmatrix} = \begin{bmatrix} R_{\text{power loss}}\\ R_{\text{wear}}\\ R_{\text{blow by}}\\ R_{\text{oil}} \end{bmatrix} = \mathbf{f}_1\left(X_1, X_2, x_3, x_4\right).
\end{aligned}
\tag{16}
$$

### 4.2.2 PATC Results

The targets for the top system performance (fuel consumption) were set as [$T^{\mu}$, $T^{\sigma}$] = [0, 0]. The means and standard deviations in Eqs. (15) and (16) were evaluated by MCS. The convergence criteria for PATC were the same as those for Example 1. For comparison, the PAIO problem was also solved using the SORA method (Du and Chen, 2004).

Two probabilistic optimization scenarios were examined. In the first scenario, the mean and standard deviation values are used in the objective functions of the PAIO (Eq. (14)) and PATC problems (Eqs. (15) and (16)) without normalization The optimal solution results using top-down and bottom-up strategies are listed in Tables 4 and 5.

**Table 4** Comparison of optimal designs (scenario 1)

| Design variable | Description | Initial point | PAIO | PATC top-down | PATC bottom-up |
|---|---|---|---|---|---|
| $\mu_{X_1}$ ($\mu m$) | Ring surface roughness | 1.0 | 4.0 | 4.0063 | 4.0 |
| $\mu_{X_2}$ ($\mu m$) | Liner surface roughness | 1.0 | 6.1193 | 6.1130 | 6.1193 |
| $x_3$ ($GP_a$) | Liner Young's modulus | 100 | 80 | 80.0445 | 80 |
| $x_4$ ($BHV$) | Liner hardness | 100 | 240 | 240 | 240 |

**Table 5** Comparison of optimal objective function values (scenario 1)

| | PAIO | PATC (top-down) | Confirmed top-down solution | PATC (bottom-up) | Confirmed bottom-up solution |
|---|---|---|---|---|---|
| $obj^*$ | 2.855e-1 | 2.8537e-1 | 2.8554e-1 | 2.8537e-1 | 2.8549e-1 |
| $\mu^*_{R_{\text{fuel}}}$ | 5.343e-1 | 5.3375e-1 | 5.3429e-1 | 5.3375e-1 | 5.3425e-1 |
| $\sigma^*_{R_{\text{fuel}}}$ | 8.391e-3 | 8.6527e-3 | 8.3825e-3 | 8.6527e-3 | 8.3911e-3 |
| $\mu^*_{R_{\text{power loss}}}$ | 3.922e-1 | 3.9175e-1 | 3.9234e-1 | 3.9159e-1 | 3.9218e-1 |
| $\sigma^*_{R_{\text{power loss}}}$ | 3.448e-2 | 3.5163e-2 | 3.4438e-2 | 3.5204e-2 | 3.4478e-2 |

As shown in Table 4, the two coordination strategies lead to the same optimal solution, which is also the same as that from the PAIO formulation. In Table 5, the optimal response moments under columns "PATC" are confirmed by substituting the optimal points back into the PAIO fully integrated analysis models for $R_{\text{fuel}}$ and $R_{\text{power loss}}$. The values of $obj^*$ were computed as $\left(\mathrm{T}^{\mu}-\mu^*_{R_{\text{fuel}}}\right)^2+\left(\mathrm{T}^{\sigma}-\sigma^*_{R_{\text{fuel}}}\right)^2$. The confirmed solutions in Table 5 are very close to those from PATC and PAIO, indicating that use of the first two moments for matching probabilistic behaviors is sufficient for this example.

The variance of fuel consumption in Table 5, $\sigma^*_{R_{\text{fuel}}}$, is very small compared to the optimal mean value $\mu^*_{R_{\text{fuel}}}$. Using non-normalized objective functions in Eq. (15) is biased towards minimizing the mean of fuel consumption. The results we obtained are meaningful because they are close to those from Kokkolaras *et al*. (2004a), which were obtained only considering the first moments of probabilistic performance.

In the second scenario, the mean and standard deviation values in the objective functions of the PAIO and PATC problem formulations were normalized. The mean and standard deviation terms of the top-level element response (fuel consumption) in $O_0$ are normalized by their best achievable values, $\mu^{\min}_{R_{\text{fuel}}}$ = 0.5359 and $\sigma^{\min}_{R_{\text{fuel}}}$ = 0.0033, respectively. Similarly, the mean and standard deviation terms of the bottom-level element response (power loss) in $O_1$ are normalized with $\mu^{\min}_{R_{\text{power loss}}}$ = 0.3916 and $\sigma^{\min}_{R_{\text{power loss}}}$ = 0.0129. The obtained optimal design and objective value are compared in Tables 6 and 7.

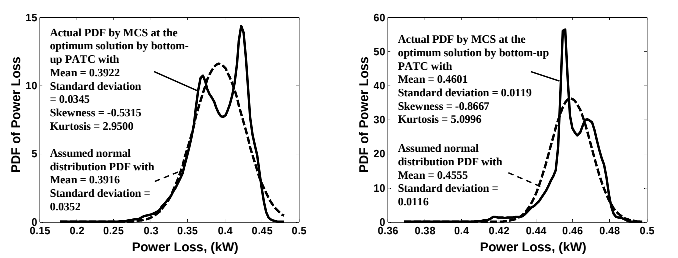
**Table 6** Comparison of optimal designs (scenario 2)

| Design variable | Description | Initial point | PAIO | PATC by top-down | PATC by bottom-up |
|---|---|---|---|---|---|
| $\mu_{X_1}$ ($\mu m$) | Ring surface roughness | 1.0 | 7.0 | 7.0 | 7.0 |
| $\mu_{X_2}$ ($\mu m$) | Liner surface roughness | 1.0 | 7.0 | 7.0 | 7.0 |
| $x_3$ ($GP_a$) | Liner Young's modulus | 100 | 340 | 340 | 340 |
| $x_4$ ($BHV$) | Liner hardness | 100 | 234.7299 | 234.7299 | 234.7299 |

**Table 7** Comparison of optimal objective function values (scenario 2)

| | PAIO | PATC (top-down) | Confirmed top-down solution | PATC (bottom-up) | Confirmed bottom-up solution |
|---|---|---|---|---|---|
| $obj^*$ | 1.9326e+0 | 1.9074e+0 | 1.9369e+0 | 1.8629e+0 | 1.9369e+0 |
| $\mu^*_{R_{\text{fuel}}}$ | 5.5131e-1 | 5.5016e-1 | 5.5132e-1 | 5.5100e-1 | 5.5132e-1 |
| $\sigma^*_{R_{\text{fuel}}}$ | 3.0855e-3 | 3.0487e-3 | 3.0931e-3 | 2.9615e-3 | 3.0931e-3 |
| $\mu^*_{R_{\text{power loss}}}$ | 4.6008e-1 | 4.5555e-1 | 4.6011e-1 | 4.5902e-1 | 4.6011e-1 |
| $\sigma^*_{R_{\text{power loss}}}$ | 1.1863e-2 | 1.1615e-2 | 1.1892e-2 | 1.1005e-2 | 1.1892e-2 |

In Scenario 2, top-down and bottom-up strategies also converged to identical optimal points and the confirmed optimal objective values are very close to those from the PAIO. Using the normalized objective functions results in emphasizing the standard deviation of fuel consumption, and the probabilistic optimization reaches a different optimal solution from that found in the first scenario. For this bi-level problem with only one element at each level, the top-down PATC converges after two cycles while the bottom-up PATC converges after one cycle.




a) PDF of $R_{\text{power loss}}$ obtained using the objective function without normalization

b) PDF of $R_{\text{power loss}}$ obtained using the normalized objective function

**Figure 5** Verification of distributions of power loss in two scenarios

The actual PDF of the power loss and the assumed normal PDF in $O_0$ are plotted in Fig. 5 for both scenarios. In both cases, the actual PDF of the power loss has two modes. However, matching the first two moments in PATC seems to be sufficient for this example and can lead to the same solution as that of the PAIO. The main reason is that the system level analysis model $f_{\text{fuel}}(R_{\text{power loss}})$ is nearly linear. Therefore the first two moments of fuel consumption are close to linear functions of the first two moments of power loss, regardless of the actual distribution of the power loss. Even for nonlinear models, it is sufficient to consider the first two moments as long as the first two moments of lower level performance dominate the uncertainty propagation on higher level responses.

Because the objective functions of the optimization problems for elements $O_0$ and $O_1$ involve multiple deviation items, we find that special care must be taken when selecting the starting point, weighting factors, and normalization technique. Different starting points should be used if local optima are suspected. For the tolerance variables ($\varepsilon^{\mu}$ and $\varepsilon^{\sigma}$), weighting factors can neither be too large nor too small. Large weights may trap the optimum at a consistent but inferior solution after the first few cycles, while small weights may cause slow convergence to a consistent solution.

## 5 CONCLUSIONS

We extended previous work on ATC under uncertainty to a more general setting. We discussed the meaning of dealing with design targets in a probabilistic framework. Following established quality engineering principles, we proposed a particular PATC formulation that matches the first two moments of random responses and linking variables with assigned targets.

An important issue related to the accuracy of the optimal solution by PATC is how many moments are sufficient to match random responses and linking variables. Based on our empirical studies using two examples, when matching the first two moments of random variables, PATC converges to the same optimal solution as PAIO under two conditions: 1) when the distributions of all matching quantities are close to normal distributions (i.e., the true mean and variance of matching quantities are close to those of assumed normal distributions, as observed in the geometric programming problem), and 2) when the first two moments to be matched have dominating impact on the optimal solution, as observed in the ring/liner problem; otherwise, PATC may lead to a different optimum with an inferior objective function value. In that case, higher-order moments may need to be matched in PATC. We also need to point out that the objective functions in PATC often involve multiple deviation items. When multiple equivalent optimal solutions exist, a situation that often happens in robust design, PATC provides the same optimal objective function value as that from PAIO, but the two approaches may reach different optimal designs. As with ATC and nonlinear optimization problems in general, local solutions may be obtained by PATC, and determining the global one presents the usual challenges.

Future research may be conducted in the following directions. First, distributions of random quantities matched in PATC are usually not known beforehand, and so it is desirable to create an efficient technique to determine when higher-order moments are necessary. Second, the number of decision variables increases in each element optimization problem under PATC when higher order moments are matched. It is desirable to investigate the impact of the order of moments on the convergence efficiency. Third, coordination strategies specific to the nature of probabilistic optimization problems will need to be investigated further to enhance convergence as computational costs will rise with increased problem size and complexity. Finally, techniques can be developed to identify multiple equivalent solutions in upper level problems in PATC so that multiple candidate targets can be used to explore the design solutions in lower level elements.

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